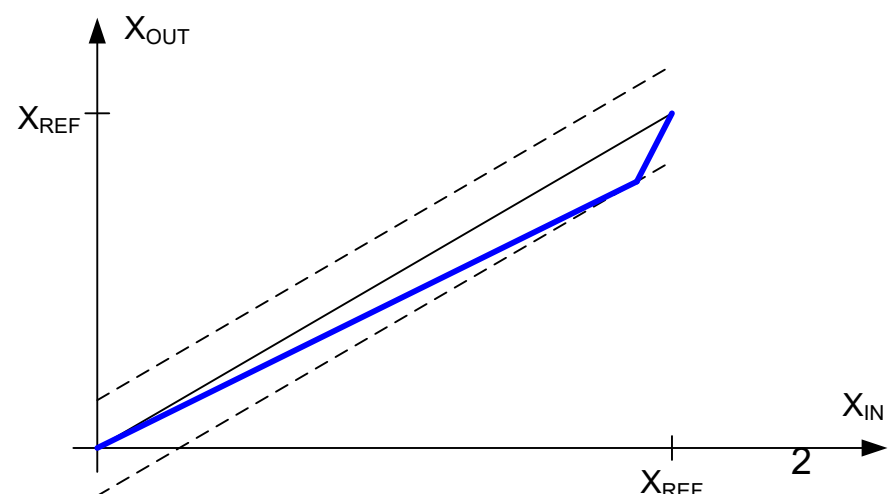# EE 435

Lecture 28

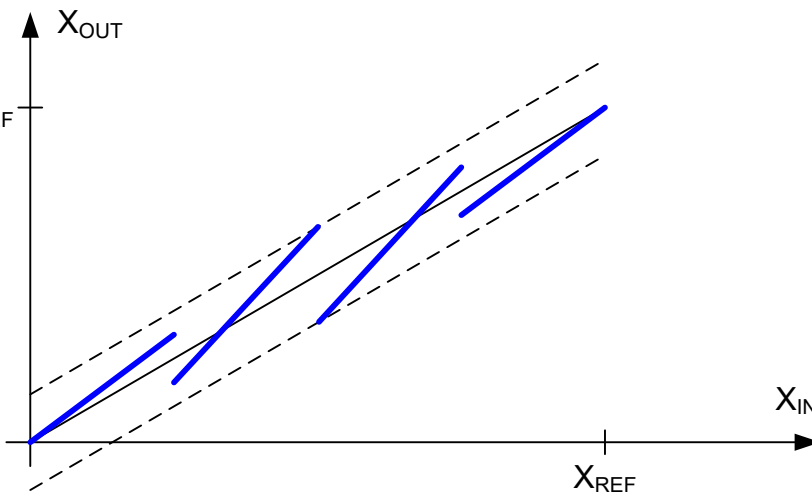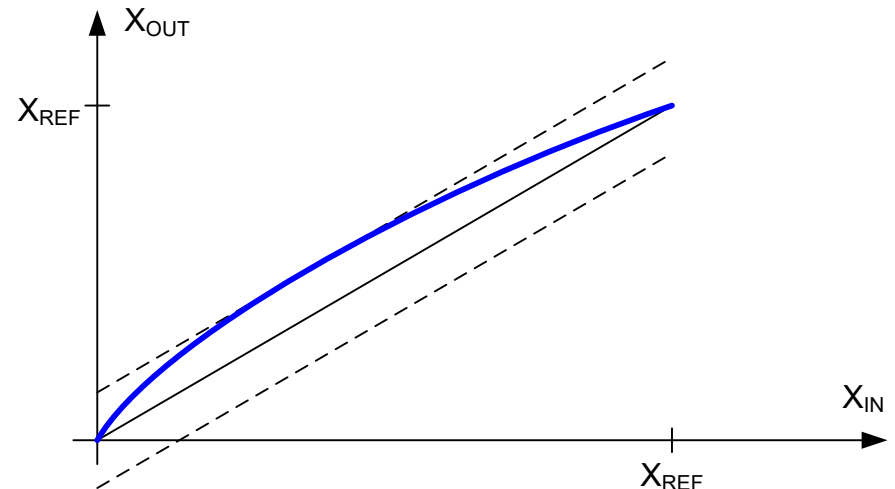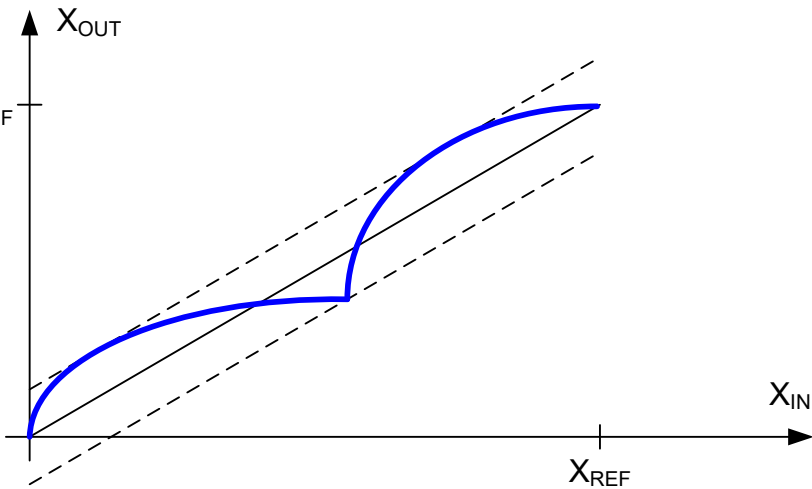## **Data Converters**

- Spectral Performance

INL Often Not a Good Measure of Linearity

Four identical INL with dramatically different linearity

2

Linearity Issues

- INL is often not adequate for predicting the linearity performance of a data converter

- Distortion (or lack thereof) is of major concern in many applications

- Distortion is generally characterized in terms of the harmonics that may appear in a waveform

Distortion Analysis

Total Harmonic Distortion, THD

$$\text{THD} = \frac{\text{RMS voltage in harmonics}}{\text{RMS voltage of fundamental}}$$
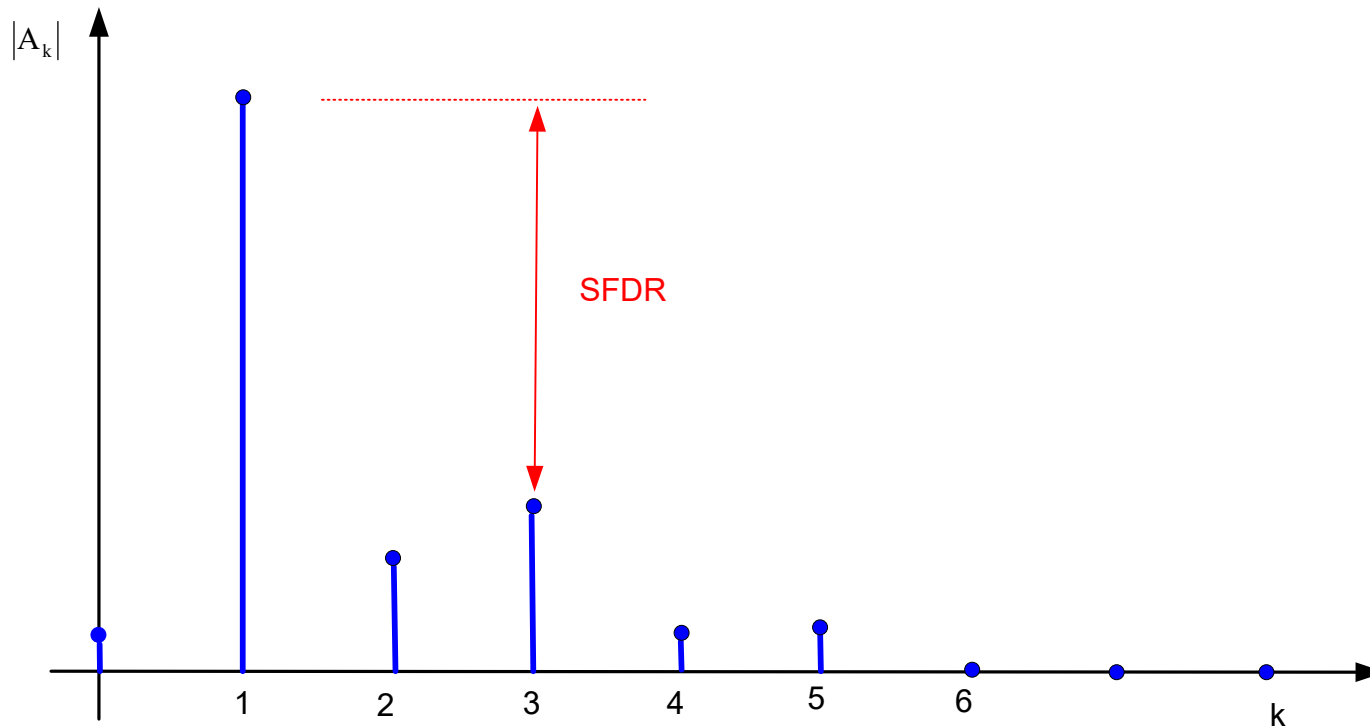
$$\text{THD} = \frac{\sqrt{\left(\frac{A_2}{\sqrt{2}}\right)^2 + \left(\frac{A_3}{\sqrt{2}}\right)^2 + \left(\frac{A_4}{\sqrt{2}}\right)^2 + ...}}{\frac{A_1}{\sqrt{2}}}$$

$$\text{THD} = \frac{\sqrt{\sum_{k=2}^{\infty} A_k^2}}{A_1}$$

4

Distortion Analysis

Spurious Free Dynamic Range, SFDR

The SFDR is the difference between the fundamental and the largest harmonic

SFDR is usually determined by either the second or third harmonic

5

Distortion Analysis

T

T_S
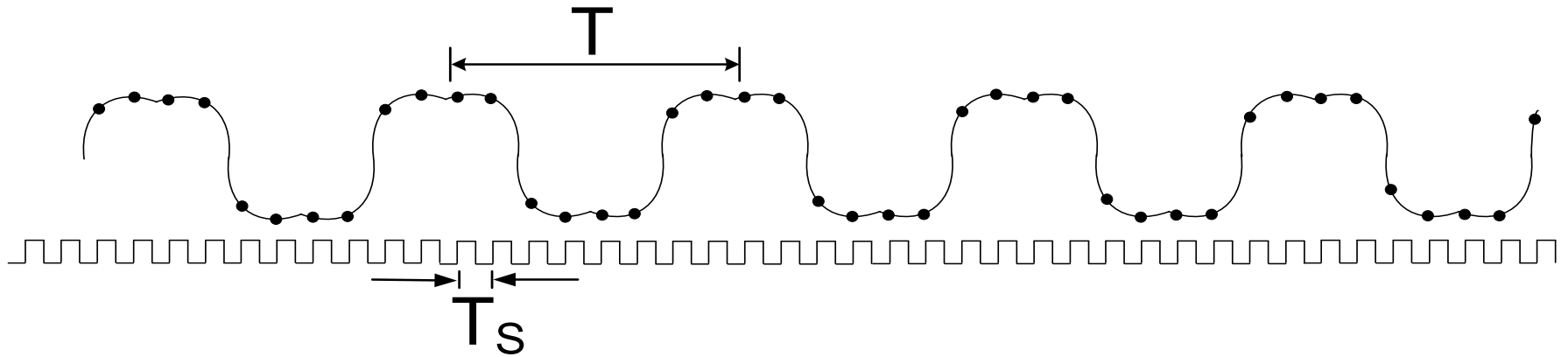
THEOREM (conceptual) : If a band-limited periodic signal is sampled at a rate that exceeds the Nyquist rate, then the Fourier Series coefficients can be directly obtained from the DFT of a sampled sequence.

$$\langle x(kT_S) \rangle_{k=0}^{N-1} \Longleftrightarrow \langle X(k) \rangle_{k=0}^{N-1}$$

Distortion Analysis
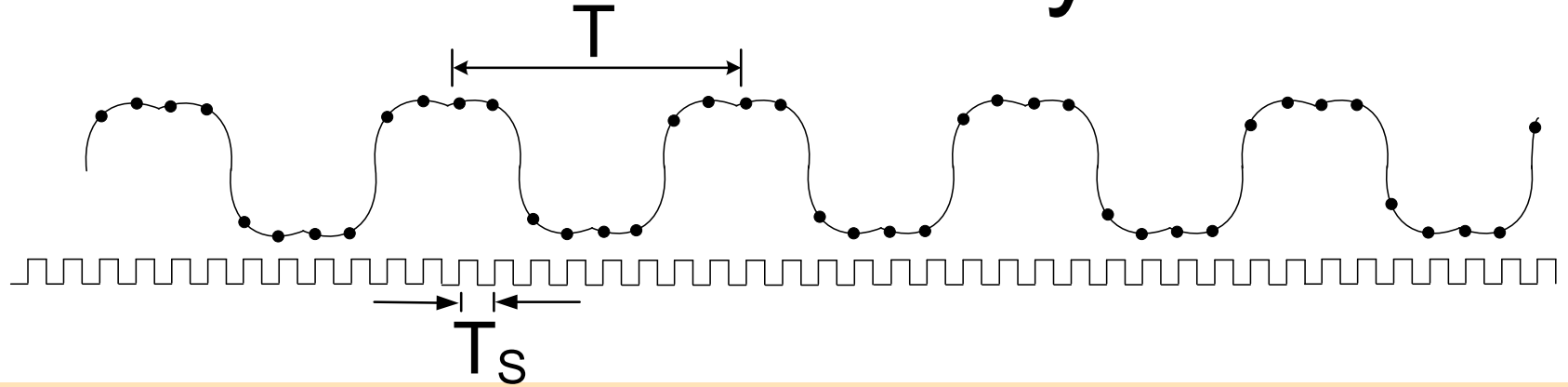

THEOREM: Consider a periodic signal with period T=1/f and sampling period $T_S=1/f_S$. If N_P is an integer, x(t) is band limited to f_{MAX}, and $f_s>2f_{max}$, then

$$\left|A_m\right| = \frac{2}{N}\left|X\left(mN_P + 1\right)\right| \qquad\qquad 0 \le m \le h - 1$$

$$X(k) = 0$$

and   for all k not defined above

where $\left\langle X(k)\right\rangle_{k=0}^{N-1}$ is the DFT of the sequence $\left\langle x\left(kT_S\right)\right\rangle_{k=0}^{N-1}$

$$h = \text{Int}\left(\frac{f_{MAX}}{f} - \frac{1}{N_P}\right)$$

N=number of samples, N_P is the number of periods, and

Note spectral components of interest as mN_p+1

Key Theorem central to Spectral Analysis that is widely used !!! and often "abused"

Why is this a Key Theorem?

THEOREM: Consider a periodic signal with period T=1/f and sampling period $T_S=1/f_S$. If N_P is an integer, x(t) is band limited to f_{MAX}, and $f_s>2f_{max}$, then

$$|A_m| = \frac{2}{N}|X(mN_P + 1)| \qquad\qquad 0 \le m \le h\text{-}1$$

$$X(k) = 0$$

and   for all k not defined above

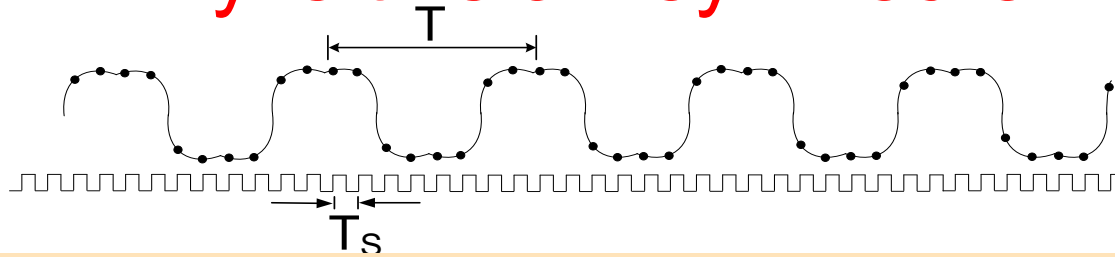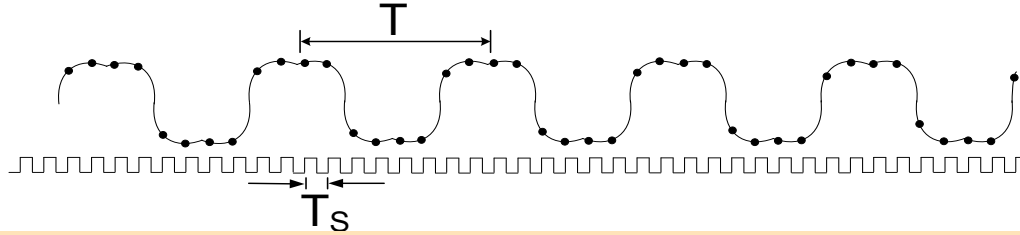where $\langle X(k) \rangle_{k=0}^{N-1}$ is the DFT of the sequence $\langle x(kT_S) \rangle_{k=0}^{N-1}$

N=number of samples, N_P is the number of periods, and $h = Int\left(\dfrac{f_{MAX}}{f} - \dfrac{1}{N_P}\right)$

- DFT requires dramatically less computation time than the integrals for obtaining Fourier Series coefficients
- Can easily determine the sampling rate (often termed the Nyquist rate) to satisfy the band limited part of the theorem
- If "signal" is output of a system (e.g. ADC or DAC), f_{MAX} is independent of f

How is this theorem abused?

THEOREM: Consider a periodic signal with period T=1/f and sampling period $T_S=1/f_S$. If N_P is an integer, x(t) is band limited to f_{MAX}, and $f_s>2f_{max}$, then

$$\left|A_m\right| = \frac{2}{N}\left|X\left(mN_P+1\right)\right| \qquad 0 \le m \le h\text{-}1$$

$$X(k)=0$$

and   for all k not defined above

where $\left\langle X(k)\right\rangle_{k=0}^{N-1}$ is the DFT of the sequence $\left\langle x\left(kT_S\right)\right\rangle_{k=0}^{N-1}$

N=number of samples, N_P is the number of periods, and $h = Int\left(\dfrac{f_{MAX}}{f} - \dfrac{1}{N_P}\right)$
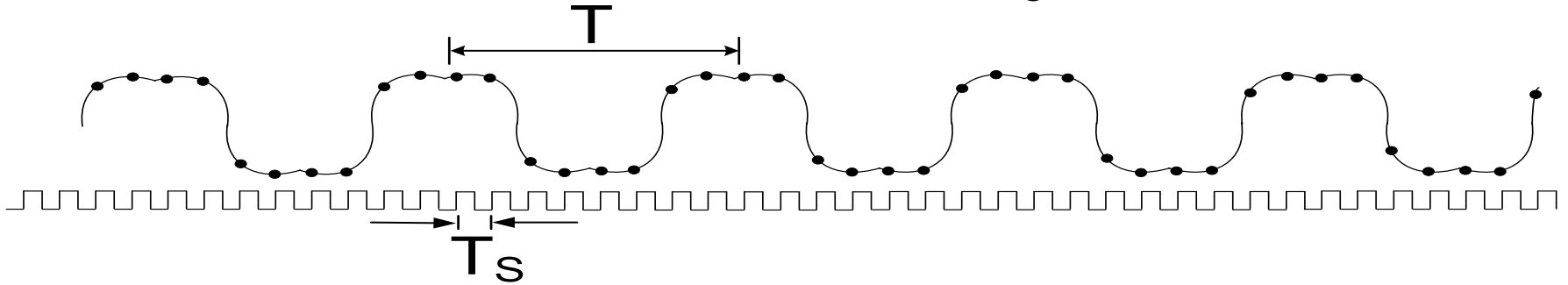
- Much evidence of engineers attempting to use the theorem when N_P is not an integer
- Challenging to have N_P an integer in practical applications
- Dramatic errors can result if there are not exactly an integer number of periods in the sampling window
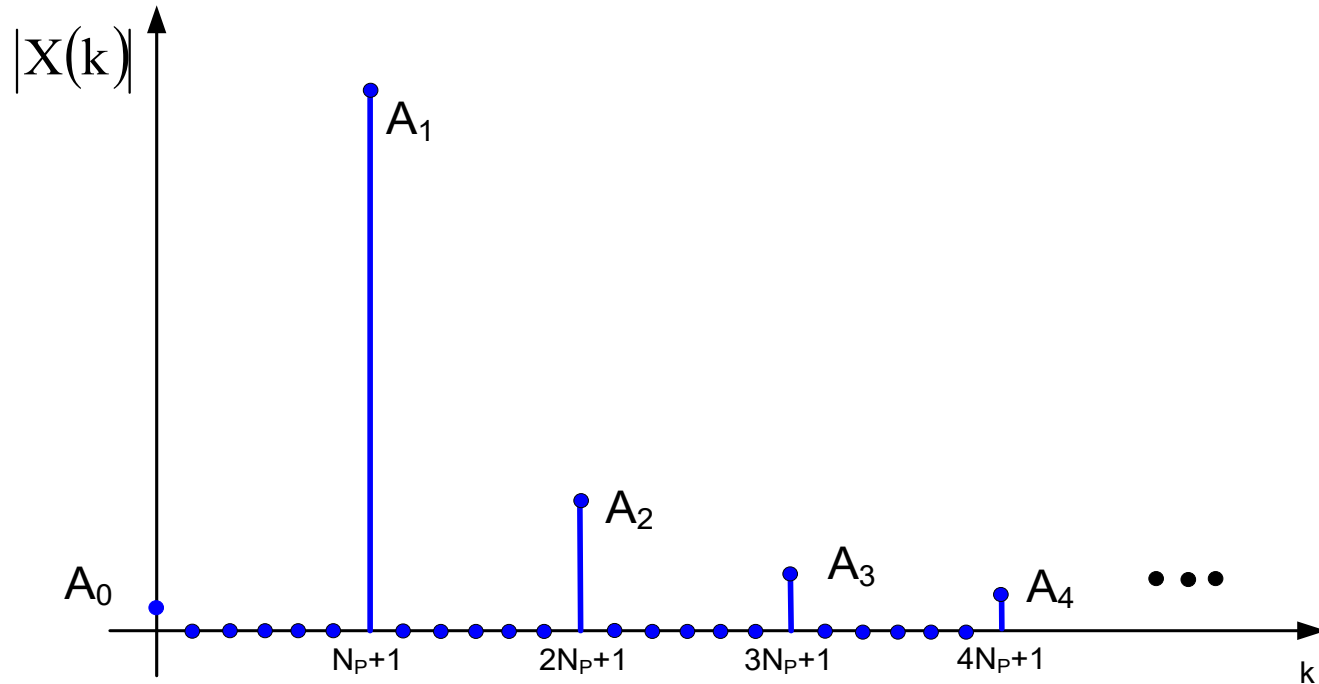
10

3 Periods of Periodic Signal in Bold Blue
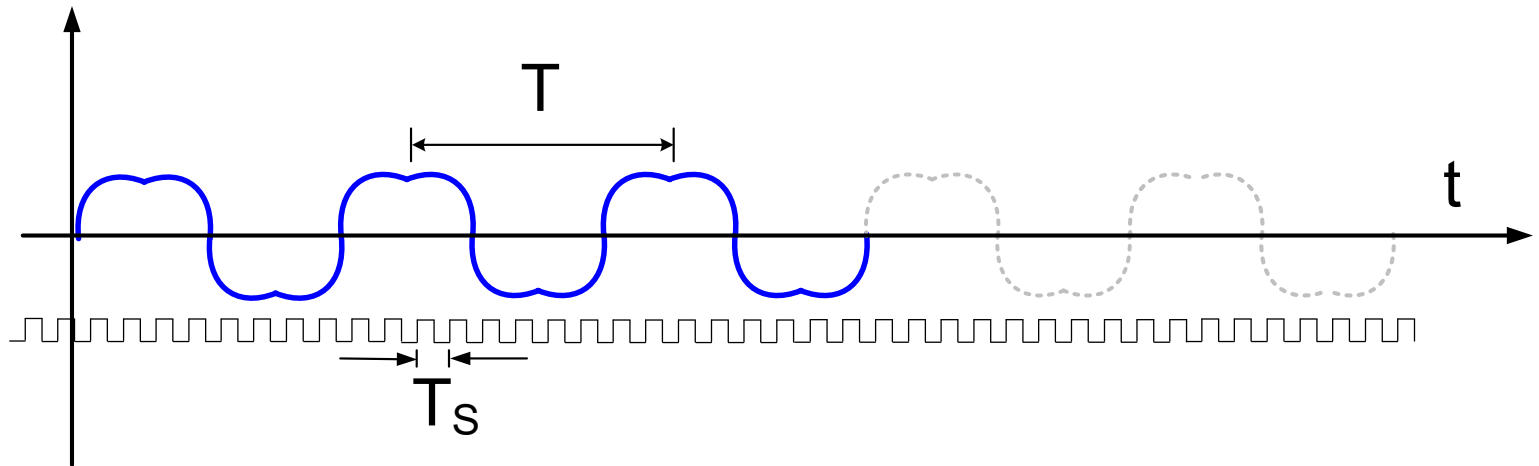
$$f_{SAMP} = f_{SIG} \frac{N}{N_P}$$

11

Distortion Analysis

If the hypothesis of the theorem are satisfied, we thus have

12

Distortion Analysis

If the hypothesis of the theorem are satisfied, we thus have
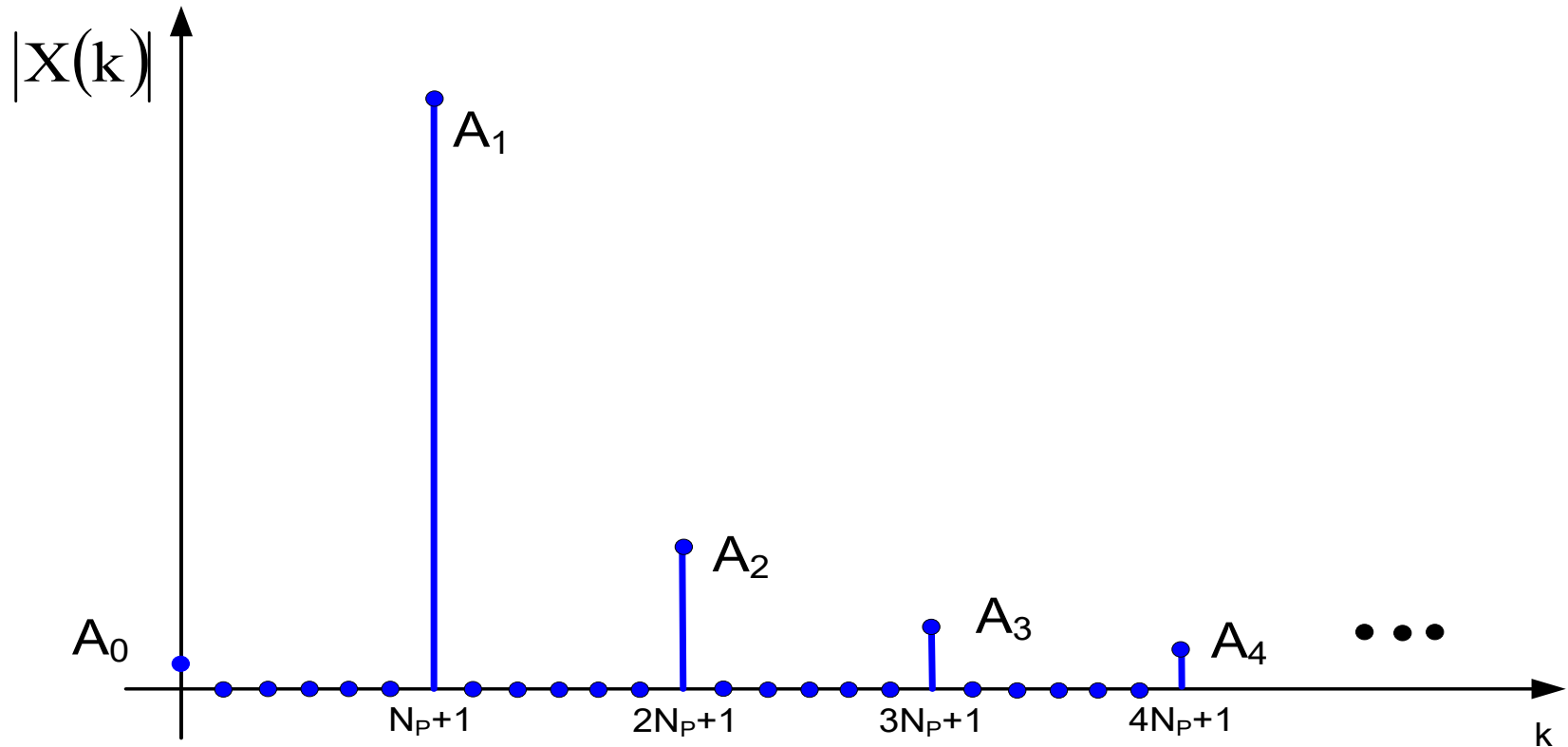
FFT is a computationally efficient way of calculating the DFT, particularly when N is a power of 2

13

DFT Examples

Recall the theorem that provided for the relationship between the DFT terms and the Fourier Series Coefficients required

1.   The sampling window be an integral number of periods

2.   The input signal is band limited to f_{MAX}

Some notation and understanding related to Fourier Series, Discrete Fourier Series, Discrete Fourier Transform, Nyquist Rate, and Nyquist Frequency may be inconsistent from source to source, confusing, and not always correctly presented in all forums

From Wikipedia – March 30 2018

Discrete Fourier series

A Fourier series is a representation of a function in terms of a summation of an infinite number of harmonically-related sinusoids with different amplitudes and phases. The amplitude and phase of a sinusoid can be combined into a single complex number, called a Fourier *coefficient*. The Fourier series is a periodic function. So it cannot represent any arbitrary function. It can represent either:

    (a) a periodic function, or

    (b) a function that is defined only over a finite-length interval; the values produced by the Fourier series outside the finite interval are irrelevant.

When the function being represented, whether finite-length or periodic, is discrete, the Fourier series coefficients are periodic, and can therefore be described by a finite set of complex numbers. That set is called a discrete Fourier transform (DFT), which is subsequently an overloaded term, because we don't know whether its (periodic) inverse transform is valid over a finite or an infinite interval. The term **discrete Fourier series (DFS)** is intended for use instead of *DFT* when the original function is periodic, defined over an infinite interval. *DFT* would then unambiguously imply only a transform whose inverse is valid over a finite interval. But we must again note that a Fourier series is a time-domain representation, not a frequency domain transform. So DFS is a potentially confusing substitute for DFT. A more technically valid description would be **DFS coefficients**.

16

Some notation and understanding related to Fourier Series, Discrete Fourier Series, Discrete Fourier Transform, Nyquist Rate, and Nyquist Frequency may be inconsistent and confusing

From Wikipedia – March 30 2018

Nyquist rate

From Wikipedia, the free encyclopedia

Not to be confused with Nyquist frequency.

This article **may be confusing or unclear** **to readers**. Please help us clarify the article. There might be a discussion about this on the talk page. *(January 2014)* *(Learn how and when to remove this template message)*

Nyquist frequency

From Wikipedia, the free encyclopedia

Not to be confused with Nyquist rate.

The **Nyquist frequency**, named after electronic engineer Harry Nyquist, is half of the sampling rate

The Nyquist frequency should not be confused with the *Nyquist rate*, which is the minimum sampling rate that satisfies the Nyquist sampling criterion for a given signal or family of signals. The Nyquist rate is twice the maximum component frequency of the function being sampled. For example, the *Nyquist rate* for the sinusoid at 0.6 f_s is 1.2 f_s, which means that at the f_s rate, it is being *undersampled*. Thus, *Nyquist rate* is a property of a continuous-time signal, whereas *Nyquist frequency* is a property of a discrete-time system.[4][5]

DFT Examples

Recall the theorem that provided for the relationship between the DFT terms and the Fourier Series Coefficients required

1. The sampling window be an integral number of periods

2. $N > \dfrac{2\,f_{max}}{f_{SIGNAL}} N_P$ (from $f_{MAX} \leq \dfrac{f}{2} \bullet \left\lceil \dfrac{N}{N_P} \right\rceil$)

Considerations for Spectral Characterization

- Tool Validation

- DFT Length and NP

- Importance of Satisfying Hypothesis

- Windowing

Considerations for Spectral Characterization

- Tool Validation (MATLAB)

- DFT Length and NP

- Importance of Satisfying Hypothesis

- Windowing

Example

WLOG assume f_{SIG}=50Hz

$$V_{IN} = \sin(\omega t) + 0.5\sin(2\omega t)$$

$$\omega = 2\pi f_{SIG}$$

$$f_{MAX\text{-}ACT} = 100 Hz$$

Consider N_P=20 N=512
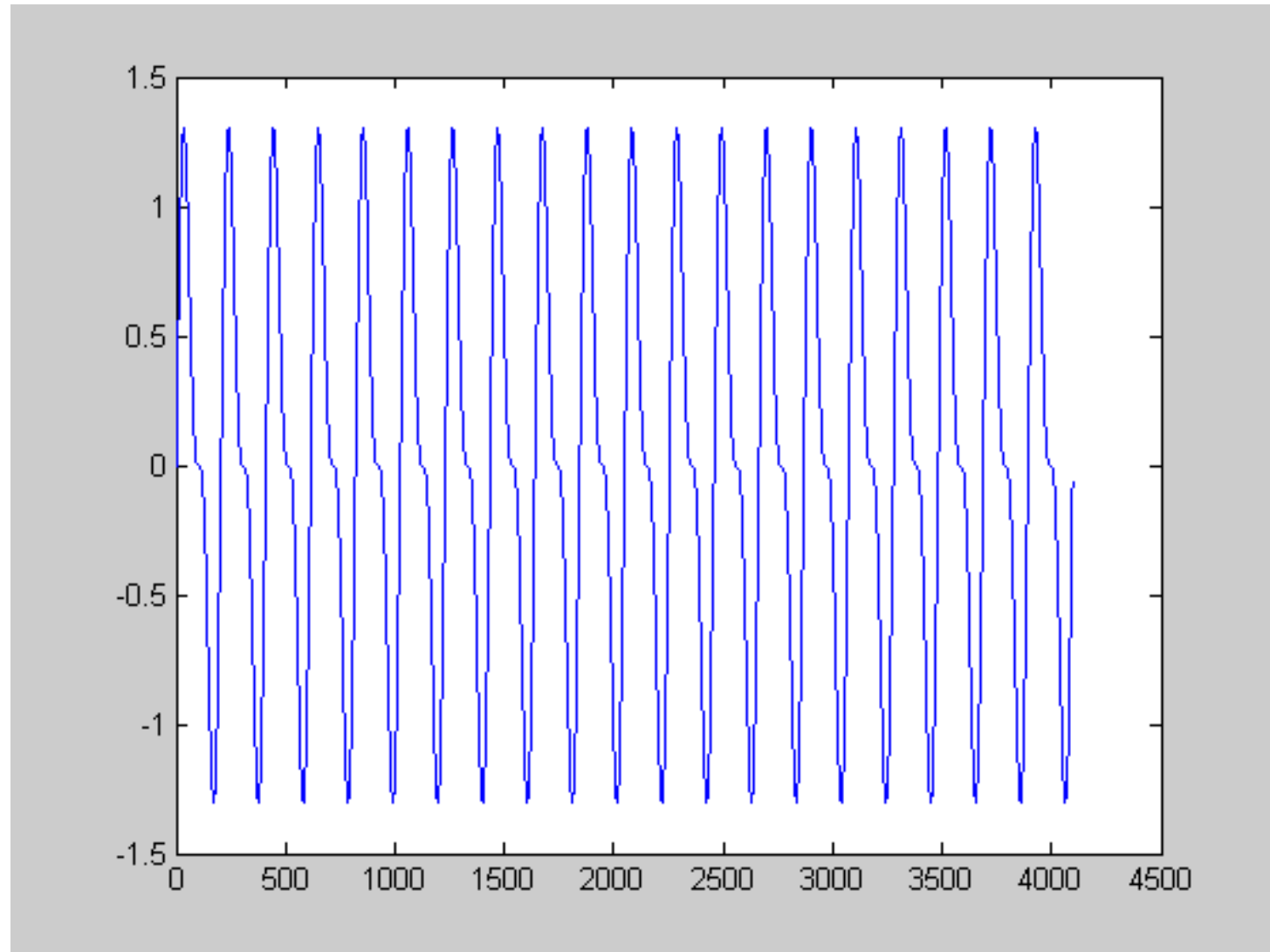
$$f_{MAX} = \frac{f_{SIG}}{2} \cdot \left[\frac{N}{N_P}\right] = \frac{50}{2} \cdot \frac{512}{20} = 640\,Hz \qquad f_{MAX\text{-}ACT} << f_{MAX}$$

$$f_{SAMPLE} = \frac{1}{T_{SAMPLE}} = \frac{1}{\left(\dfrac{N_P \cdot T_{SIG}}{N}\right)} = \left[\frac{N}{N_P}\right] f_{SIG} = 2 f_{MAX} = 1280\,Hz$$
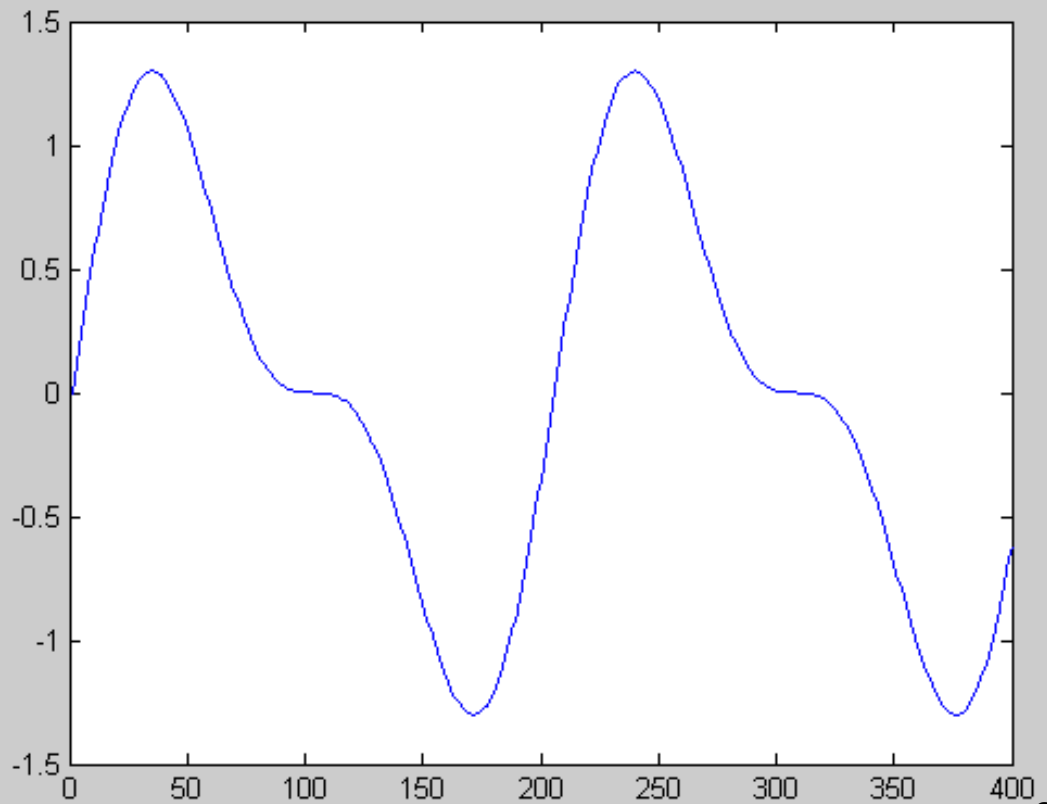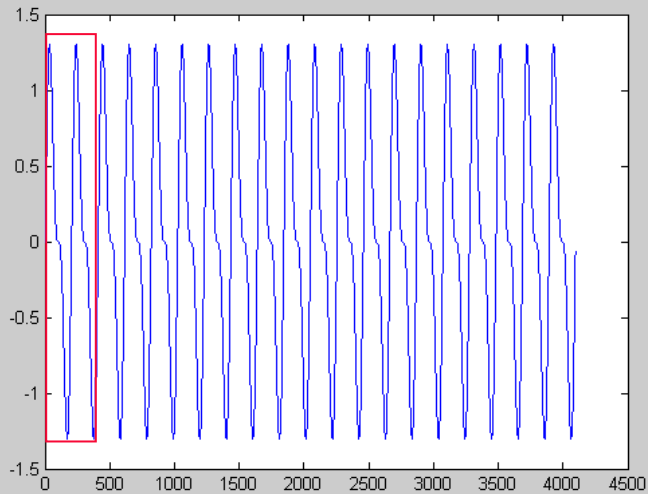
Recall $20\log_{10}(1.0) = 0.0000000$

Recall $20\log_{10}(0.5) = -6.0205999$

22

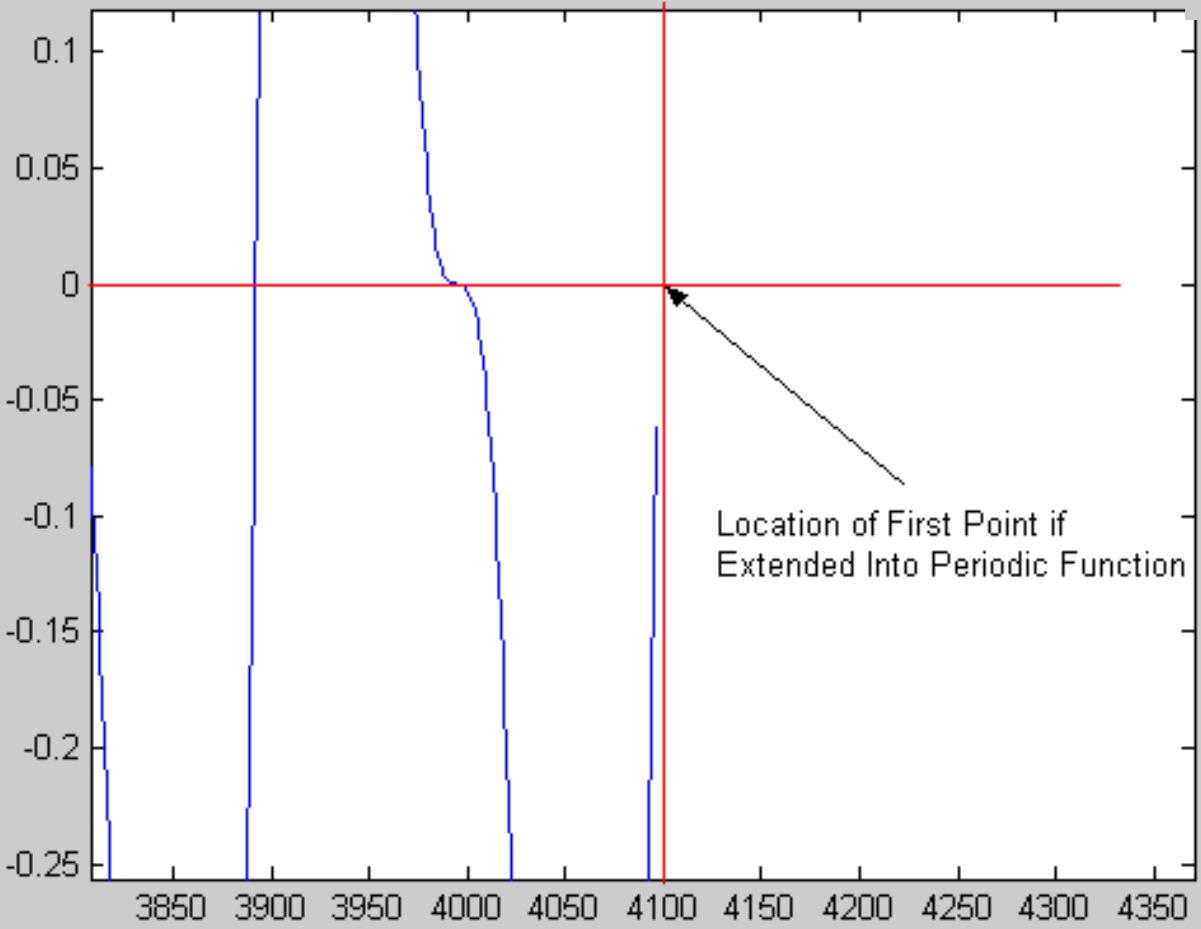# Input Waveform

23

Input Waveform

24

Input Waveform

Location of First Point if
Extended Into Periodic Function

25

Spectral Response (expressed in dB)

Rect. Window N=512 Np =20

(Actually Stem plots but points connected in plotting program)

Mag(dB) vs Frequency
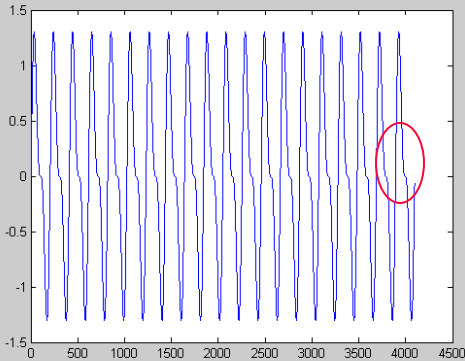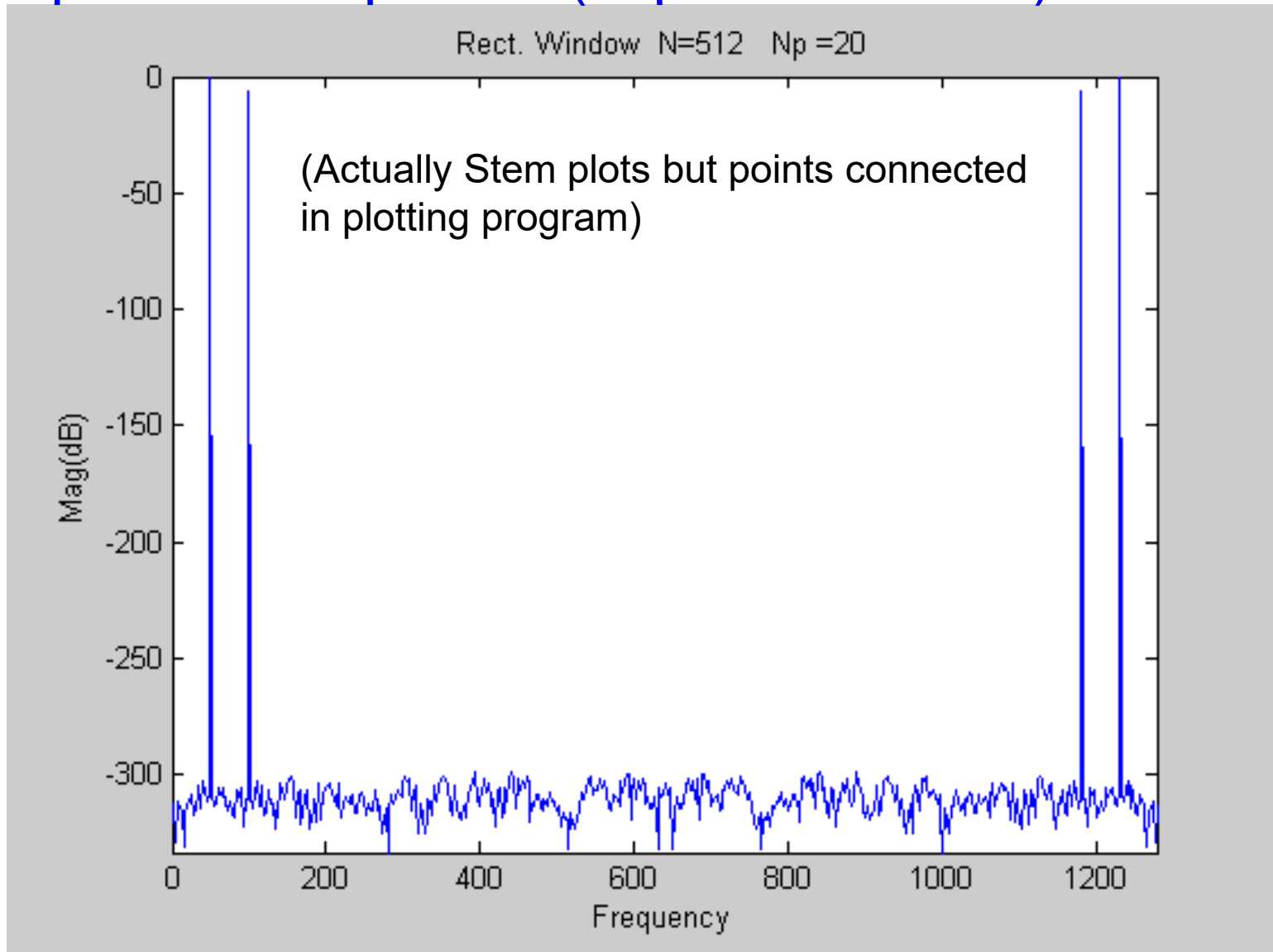
(Horizontal axis is the "Index" axis but converted to frequency)

$$f_{AXIS} = f_{SIGNAL} \frac{n-1}{N_P}$$

Spectral Response (expressed in dB)

Rect. Window N=512   Np =20

(Actually Stem plots but points connected in plotting program)

Mag(dB)
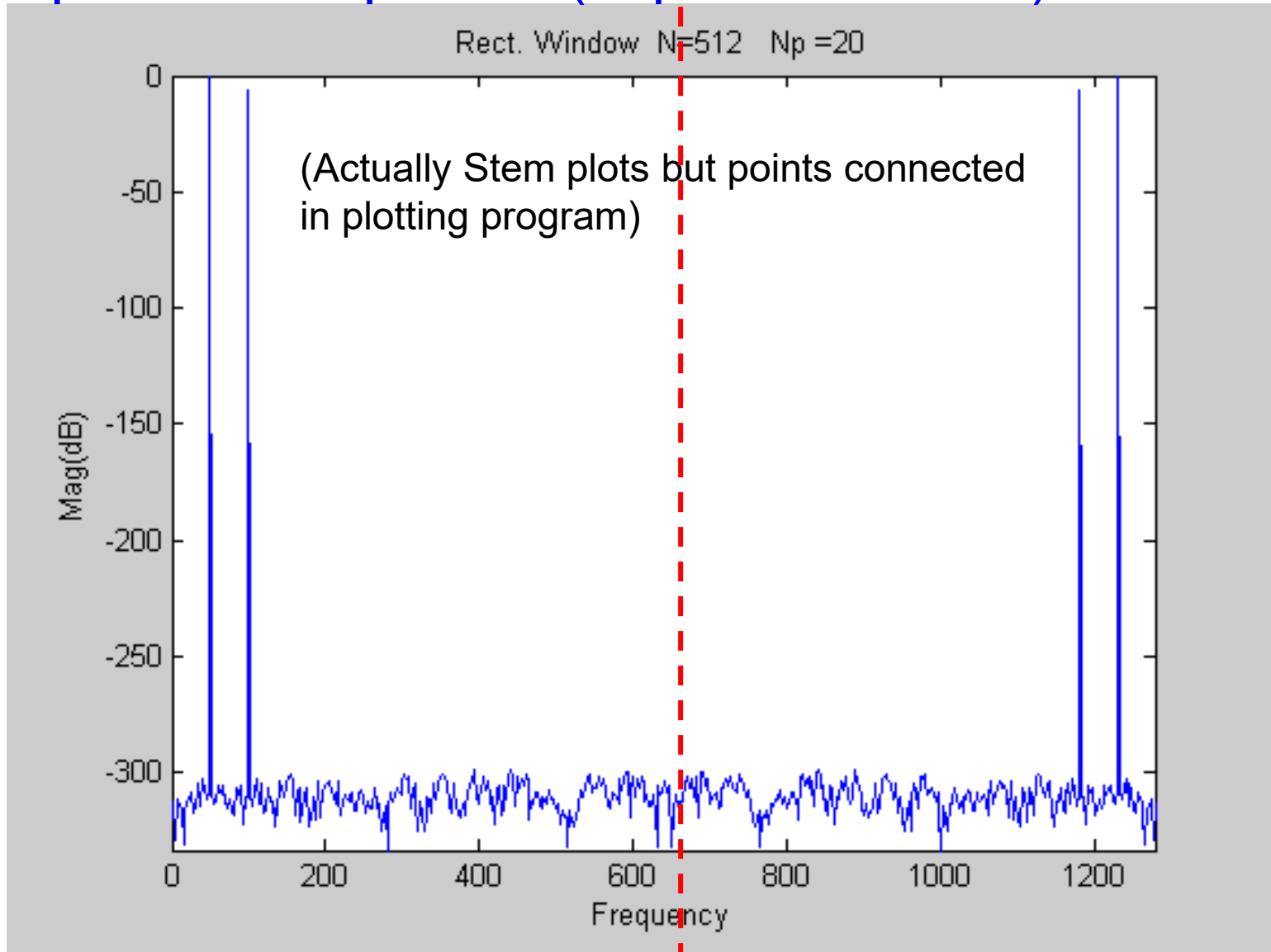
Frequency

Note Magnitude is Symmetric wrt f_{SAMPLE}

$$f_{AXIS} = f_{SIGNAL} \frac{n - 1}{N_P}$$

Spectral Response

Rect. Window N=512 Np =20
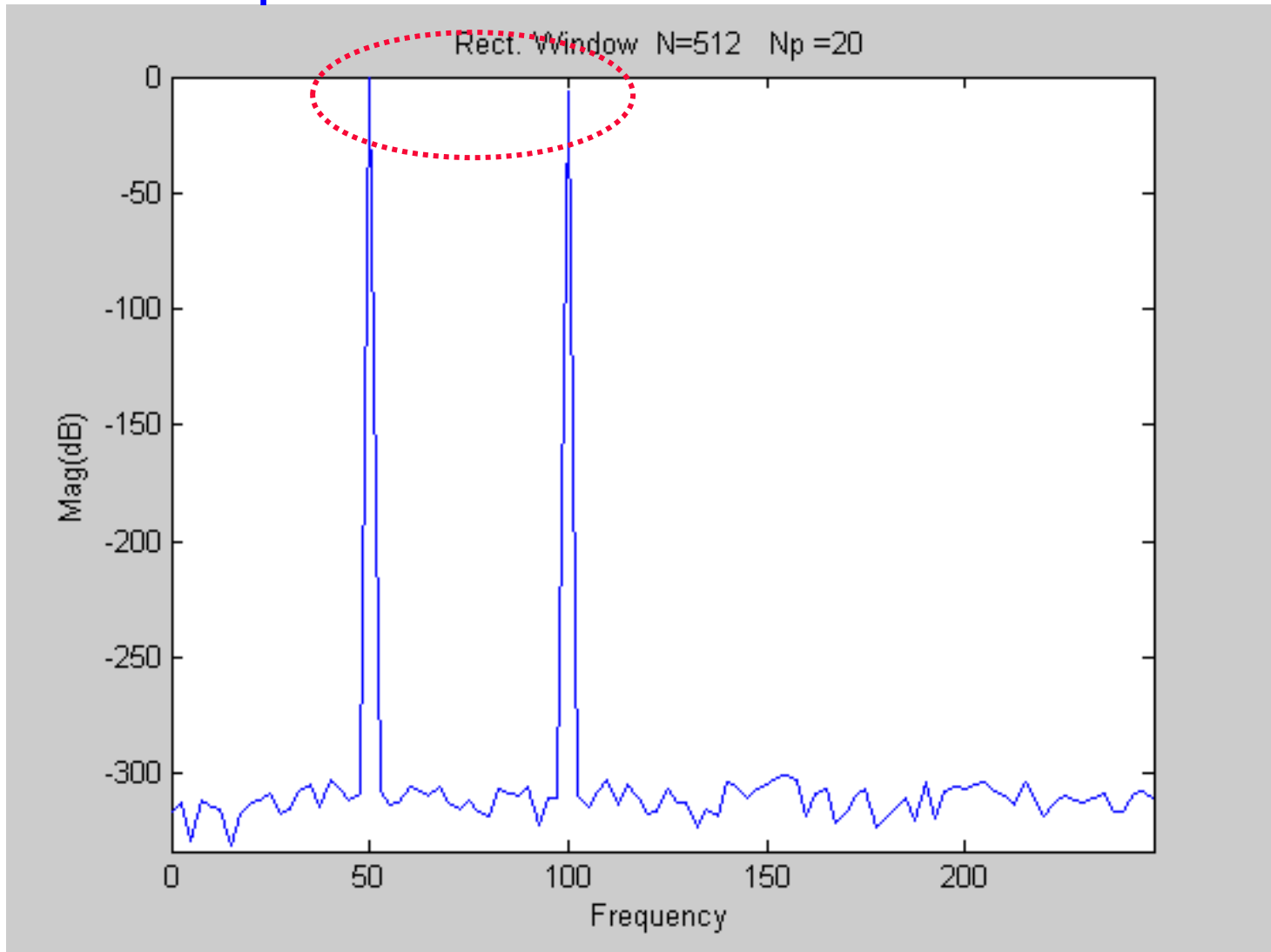
DFT Horizontal Axis Converter to Frequency : $\quad f_{AXIS} = f_{SIGNAL} \dfrac{n-1}{N_P}$

28

Spectral Response
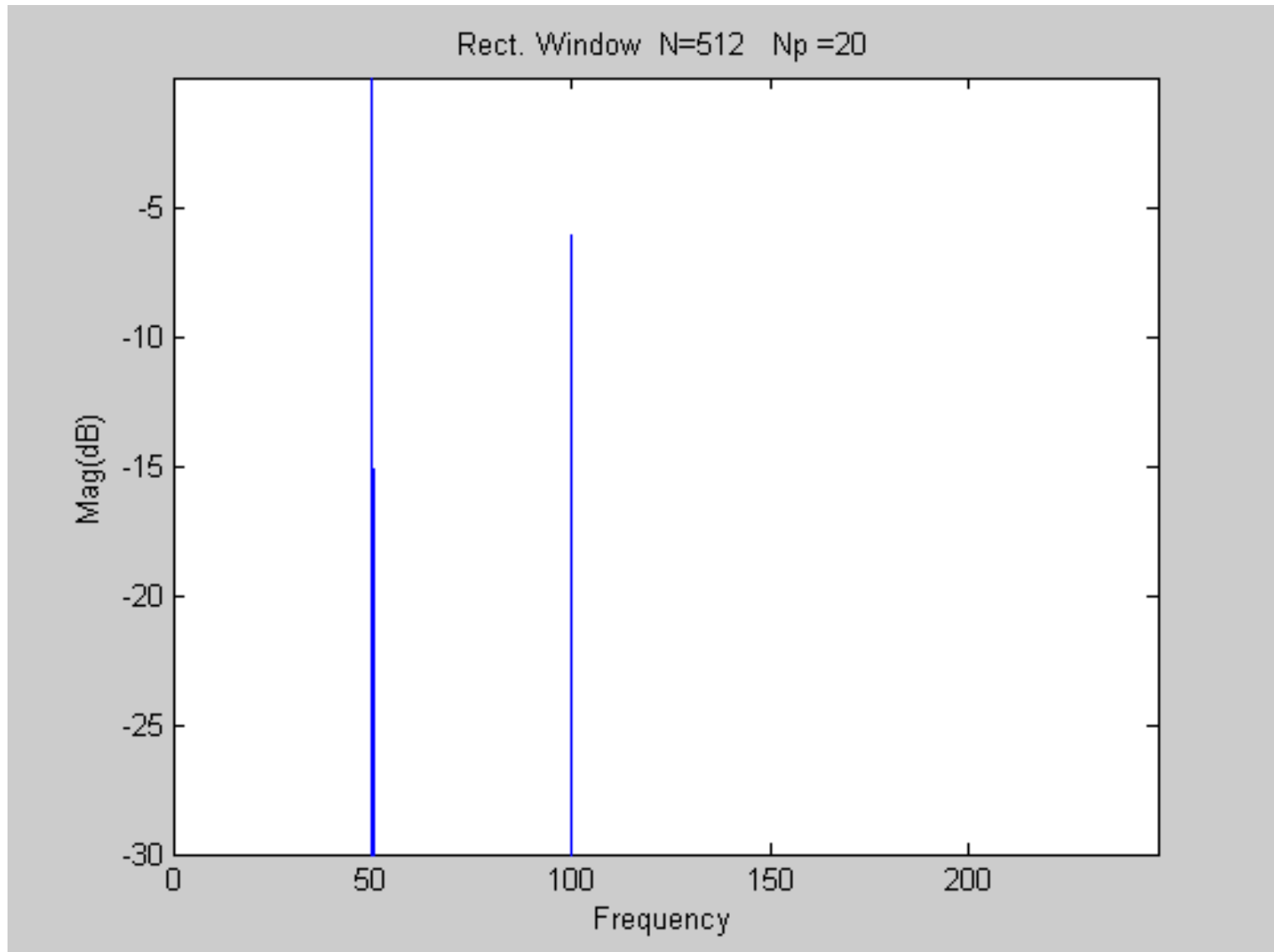
Fundamental will appear at position 1+Np = 21

Columns 1 through 5

-316.1458 -312.9517 -329.5203 -311.1473 -314.2615

Columns 6 through 10

-315.2584 -330.6258 -317.2896 -312.2316 -311.6335

Columns 11 through 15

-308.2339 -317.7064 -315.3135 -307.9349 -304.5641

Columns 16 through 20

-314.0088 -302.6391 -306.6650 -311.3733 -308.3689

Columns 21 through 25

-0.0000 -307.7012 -312.9902 -312.8737 -305.4320

Observe system noise floor due to both spectral limitations of signal generator and numerical limitations in FFT are below -300db

30

Second Harmonic at 1+2Np = 41

Columns 26 through 30

-307.8301 -309.0737 -305.8503 -312.2772 -315.7544

Columns 31 through 35

-311.9316 -316.0581 -318.3454 -306.4977 -308.6679

Columns 36 through 40

-309.9702 -305.9809 -322.1270 -310.6723 -310.3506

Columns 41 through 45

-6.0206   -309.6071 -314.1026 -307.6405 -302.9277

Columns 46 through 50

-313.0745 -304.2330 -310.8487 -317.7966 -316.3385

Third Harmonic at 1+3Np = 61

Columns 51 through 55

-307.0529 -312.7787 -312.9340 -323.2969 -314.9297

Columns 56 through 60

-318.7605 -303.5929 -305.2994 -310.6430 -306.7613

Columns 61 through 65

-304.8298 -301.4463 -301.1410 -303.1784 -317.8343

Columns 66 through 70

-308.6310 -307.0135 -321.6015 -316.6548 -309.8946

Columns 71 through 75

-306.3472 -323.0110 -319.3267 -314.7873 -310.4085

Fourth Harmonic at 1+4Np = 81

Columns 76 through 80

 -319.8926 -303.3641 -319.6263 -307.6894 -305.1945

  Columns 81 through 85

-306.8190 -304.8860 -303.6531 -307.2090 -309.8014

  Columns 86 through 90

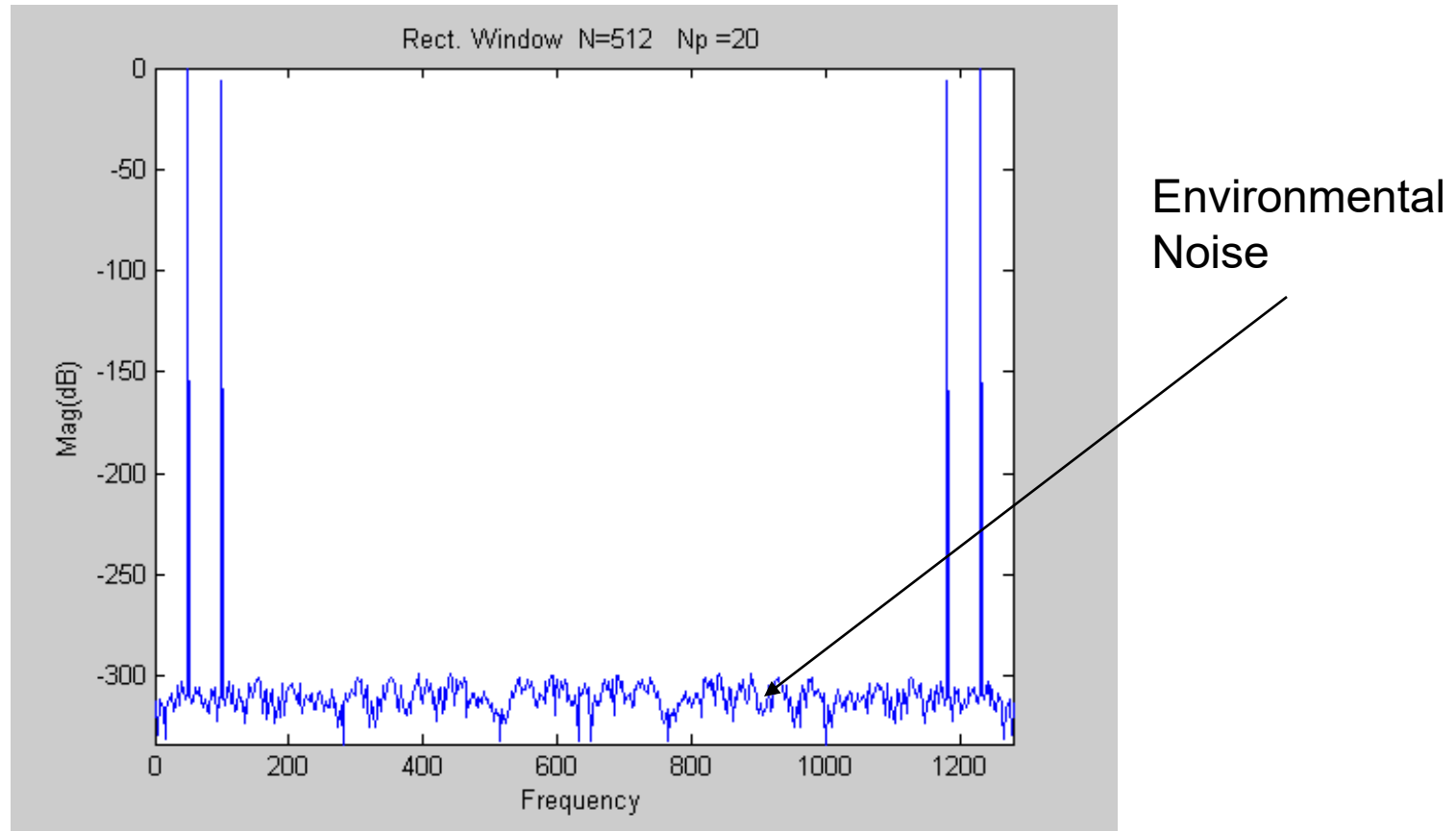-313.4988 -303.4513 -310.4969 -317.9652 -312.5846

  Columns 91 through 95

-309.8121 -311.6403 -312.8374 -310.5414 -308.7807

  Columns 96 through 100

-316.7549 -316.3395 -308.4113 -307.3766 -311.0358
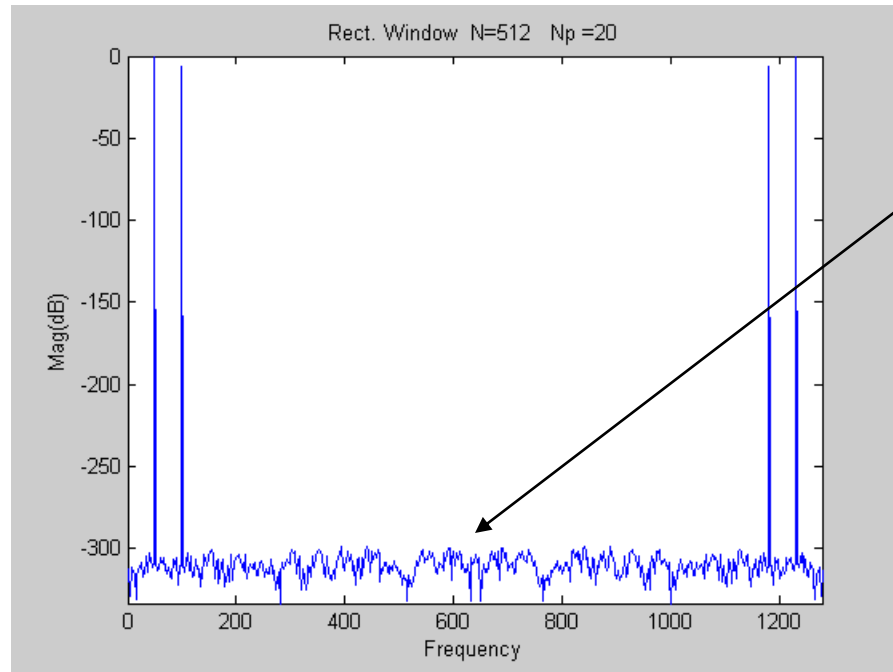
Question: How much noise is in the computational environment?

Is this due to quantization in the computational environment or to numerical rounding in the FFT?

Question: How much noise is in the computational environment?

Environmental Noise

Observation: This noise is nearly uniformly distributed
The level of this noise at each component is around -310dB

Question: How much noise is in the computational environment?

Assume A_k = -310 dB for $0 \le k \le N$

$A_{kdB} = 20 \log 10 A_k$ $\Longrightarrow$ $A_k = 10^{\frac{A_{kDB}}{20}}$

$$A_k \cong 10^{\frac{-310}{20}} = 10^{-15.5} \overset{defn}{=} \overline{A}$$

$$V_{Noise,RMS} \cong \sqrt{\sum_{k=1}^{N-1} \left(\frac{A_k}{\sqrt{2}}\right)^2} \overset{\substack{A_k = \overline{A} \\ N \text{ large}}}{=} \overline{A}\sqrt{\frac{N}{2}}$$

$$V_{Noise,RMS} \cong \overline{A}\sqrt{\frac{N}{2}} = 10^{-15.5}\sqrt{\frac{512}{2}} = 5.1 \bullet 10^{-15} \cong 5fV$$

This computational environment has a very low total computational noise and does not become significant until the 46-bit resolution level is reached !!

36

Considerations for Spectral Characterization

- Tool Validation

- DFT Length and NP

- Importance of Satisfying Hypothesis

- Windowing
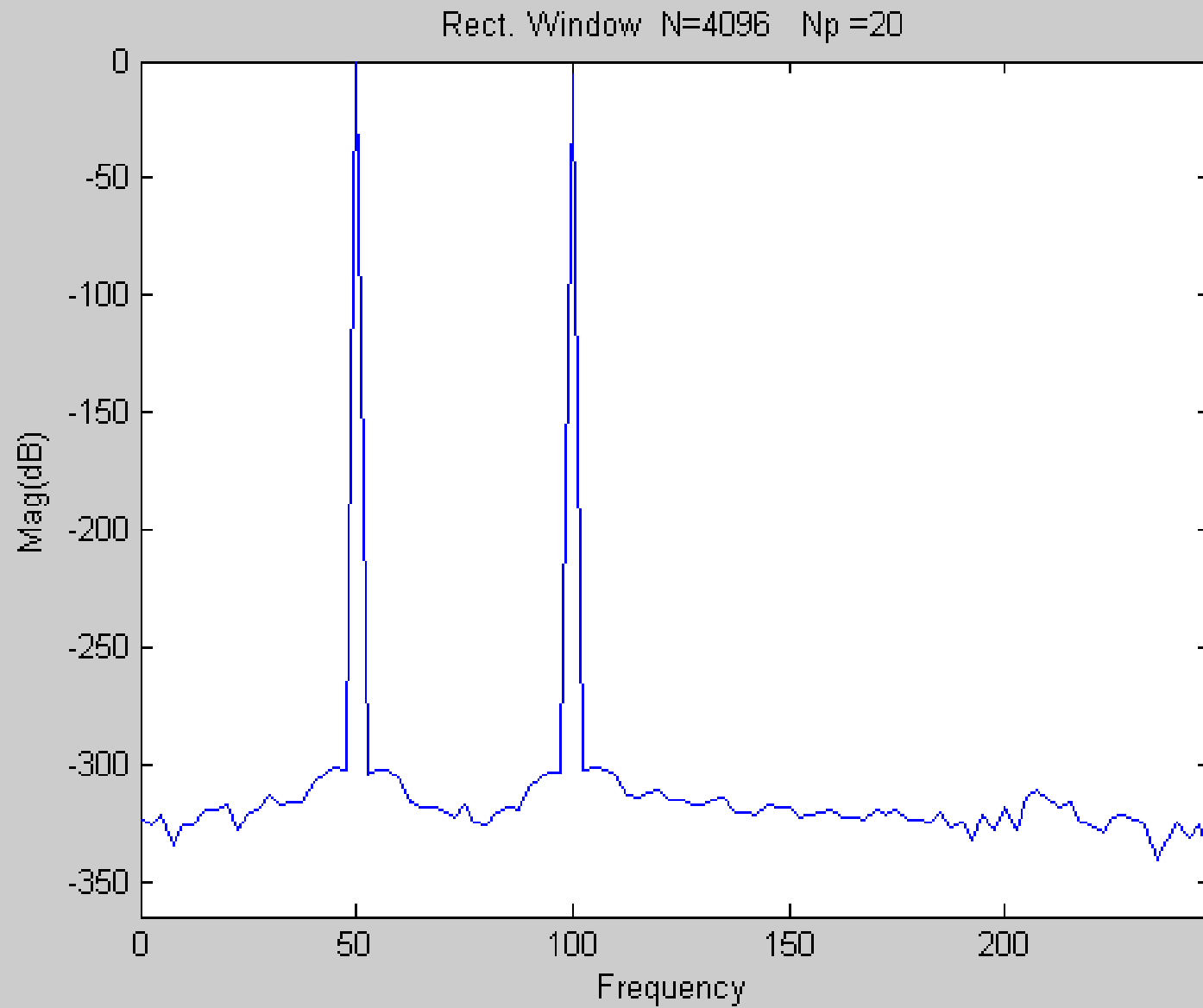
Example - Increase DFT length from 512 to 4096

WLOG assume f_{SIG}=50Hz

$$V_{IN} = \sin(\omega t) + 0.5\sin(2\omega t)$$

$$\omega = 2\pi f_{SIG}$$

Consider N_P=20 N=4096

Spectral Response

Rect. Window   N=4096   Np =20

39

Fundamental will appear at position 1+Np = 21

Columns 1 through 7

-323.9398 -325.5694 -321.3915 -334.6680 -325.2463 -325.3391 -319.3569

Columns 8 through 14

-319.7032 -317.4419 -327.4933 -321.1968 -318.2241 -312.7300 -316.8359

Columns 15 through 21

-315.5166 -316.1801 -307.8072 -304.3414 -301.3326 -301.7993 | 0 |

Columns 22 through 28

-303.9863 -302.2114 -302.5485 -306.5542 -315.4995 -318.3911 -318.4441

Columns 29 through 35

-318.7570 -322.6054 -317.3667 -324.0324 -325.8546 -320.3611 -317.8960

kth harmonic will appear at position 1+k•Np

Columns 36 through 42

-319.0051 -309.4219 -305.5698 -302.8625 -303.2207   -6.0206 -302.3437

Columns 43 through 49

-300.8222 -301.6722 -304.8150 -313.0288 -313.5963 -312.1136 -310.7740

Columns 50 through 56

-314.7706 -315.3607 -317.0331 -316.8648 -314.4965 -314.3096 -320.4308

Columns 57 through 63

-320.2843 -320.9910 -316.8320 -318.3531 -318.4341 -322.1619 -321.6183

Columns 64 through 70

-320.6985 -319.0630 -322.1485 -322.3338 -323.6365 -319.0865 -321.0791
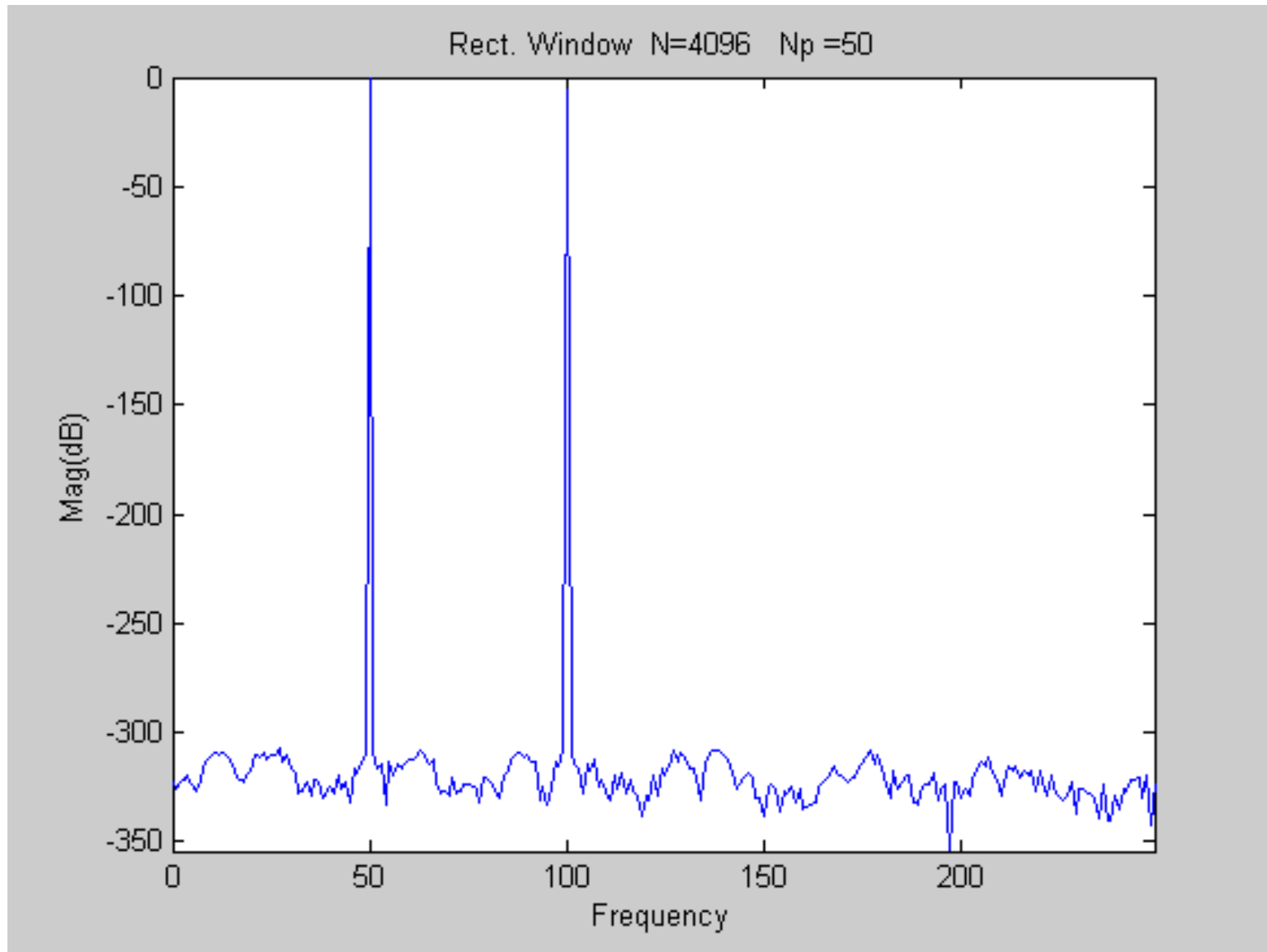
Example — Increase NP from 20 to 50

WLOG assume f_{SIG}=50Hz

$$V_{IN} = \sin(\omega t) + 0.5\sin(2\omega t)$$

$$\omega = 2\pi f_{SIG}$$

Consider N_P=50 N=4096

Spectral Response

Rect. Window   N=4096   Np =50

Fundamental will appear at position 1+Np = 51

Columns 1 through 7

-322.4309 -325.5445 -322.2645 -321.6226 -319.5894 -323.4895 -327.3216

 Columns 8 through 14

-321.2981 -316.1855 -312.3071 -310.4889 -309.6790 -309.9436 -309.3734

 Columns 15 through 21

-311.4435 -314.7665 -317.1248 -321.7733 -323.0602 -318.2119 -317.4601

 Columns 22 through 28

-310.1735 -311.1633 -308.9079 -312.0709 -310.6683 -310.6908 -307.6761

 Columns 29 through 35

-312.9440 -310.5706 -316.2098 -318.9565 -327.6885 -326.4021 -322.3135

Fundamental will appear at position 1+Np = 51

 Columns 36 through 42

-328.5059 -321.5592 -322.6183 -330.2002 -328.5051 -324.3480 -328.0173

 Columns 43 through 49

-319.3974 -325.8498 -323.1539 -331.9531 -317.0166 -318.3041 -314.9011

 Columns 50 through 56

-309.5231   0   -308.8842 -316.1343 -314.5406 -333.4024 -313.7342

 Columns 57 through 63

-319.6023 -314.9029 -316.6932 -314.7123 -311.9567 -312.0200 -309.8825

 Columns 64 through 70

-308.7103 -309.8064 -314.9393 -312.4610 -322.7229 -328.0350 -326.6767

k^{th} harmonic will appear at position 1+k•Np

Columns 71 through 77

-329.1687 -321.1102 -328.3790 -326.9774 -323.4227 -323.3388 -325.1652

Columns 78 through 84

-325.3417 -332.1905 -320.4431 -322.1461 -323.8993 -325.4370 -329.8160

Columns 85 through 91

-319.1702 -317.1792 -312.4734 -310.2585 -309.5426 -310.8963 -310.6955

Columns 92 through 98

-313.6855 -313.3882 -330.4962 -324.4762 -333.2237 -325.8694 -313.9127

Columns 99 through 105

-315.4869 -308.6364 -6.0206 -309.2723 -314.4098 -316.3311 -328.2626

k^{th} harmonic will appear at position $1+k \cdot Np$

Columns 106 through 112

-314.3378 -317.7599 -312.1738 -324.4699 -321.7568 -326.3796 -331.0818

Columns 113 through 119

-319.9292 -325.4840 -318.0998 -328.0000 -321.7632 -326.5097 -328.5867

Columns 120 through 126

-338.0360 -328.6163 -330.5881 -319.7260 -329.2289 -316.3840 -319.1143

Columns 127 through 133

-315.0684 -308.6315 -312.9640 -309.5056 -311.6251 -316.1369 -316.1064

Columns 134 through 140

-320.4989 -331.2686 -314.3479 -310.0891 -308.0023 -308.1556 -309.0616

k^{th} harmonic will appear at position $1+k\cdot Np$

Columns 141 through 147

-311.2372 -312.6180 -319.0565 -325.6750 -323.7759 -320.7444 -318.0752

Columns 148 through 154

-320.5965 -330.3083 -330.2507 -338.2118 -325.0839 -323.5993 -326.2350

Columns 155 through 161

-336.0163 -326.5945 -327.9587 -324.7636 -332.5650 -326.1828 -334.9208

Columns 162 through 168

-333.9169 -333.3995 -332.0925 -324.3599 -322.9393 -320.4507 -317.7706

Columns 169 through 175

-315.9825 -319.2534 -320.8277 -322.3018 -321.6497 -320.4065 -315.4057

Considerations for Spectral Characterization

Quantization Noise

It will be shown that the quantization that takes place in either an ADC or a DAC acts like noise and is nearly uniformly distributed in <u>all</u> DFT bins.

Thus the deviations in output of data converters caused by magnitude quantization is termed quantization noise

It will be shown later that the RMS value of the quantization noise is given by the expression

$$E_{QUANT} \cong \frac{X_{LSB}}{\sqrt{12}} = \frac{X_{REF}}{\sqrt{3} \bullet 2^{n+1}}$$

Quantization noise components in DFT bins are much larger than the computational noise which is also nearly uniformly distributed in all DFT bins

Considerations for Spectral Characterization

DFT Length and NP

- DFT Length and NP do not affect the computational noise floor

- Although not shown here yet, DFT length does reduce the <u>quantization</u> noise floor coefficients but not total quantization noise

If we assume E_{QUANT} is fixed and no signal present

$$E_{QUANT} \cong \sqrt{\sum_{k=1}^{2^{n_{DFT}}} A_k^2}$$

(these are now the DFT coefficients due to quantization noise, not computation noise)

If the A_k's are constant and equal

$$E_{QUANT} \cong A_k 2^{n_{DFT}/2}$$

Solving for A_k, obtain

$$A_k \cong \frac{E_{QUANT}}{2^{n_{DFT}/2}}$$

If input is full-scale sinusoid with only amplitude quantization with n-bit res,

$$E_{QUANT} \cong \frac{X_{LSB}}{\sqrt{12}} = \frac{X_{REF}}{\sqrt{3} \bullet 2^{n+1}}$$

(this expression is actually independent of input waveform)

Considerations for Spectral Characterization

DFT Length

$$E_{QUANT} \cong \frac{X_{LSB}}{\sqrt{12}} = \frac{X_{REF}}{\sqrt{3} \bullet 2^{n+1}}$$

Substituting for E_{QUANT}, obtain

$$A_k \cong \frac{X_{REF}}{\sqrt{3} \bullet 2^{n+1} 2^{n_{DFT}/2}}$$

This value for A_k thus decreases with the length of the DFT sampline window

Example: if n=16, n_{DFT}=12 (4096 pt transform), and X_{REF}=1V, then A_k=6.9E-8V (-143dB),

(Note A_k>> computational noise floor (-310dB for Matlab) for all practical n, n_{DFT})

Considerations for Spectral Characterization

FFT Length

If a periodic signal is present, there will be a few spectral terms in the DFT that represent the signal energy. If coherently sampled, these terms will be at the fundamental frequency and harmonics of the fundamental.

Each term in the DFT will include any harmonic content along with a quantization noise component but at the fundamental and harmonics the DFT terms will usually completely dominate the quantization noise part

Under these assumptions, if the A_k's due to quantization noise are constant and equal

$$E_{QUANT} \cong \sqrt{\sum_{\substack{k=1 \\ k \neq harmonic}}^{2^{n_{DFT}}} A_k^2} \quad \longrightarrow \quad E_{QUANT} \cong A_k 2^{n_{DFT}/2}$$

Solving for A_k, we still obtain for DFT terms that are not harmonic terms

$$A_k \cong \frac{E_{QUANT}}{2^{n_{DFT}/2}}$$

Considerations for Spectral Characterization

- Tool Validation

- DFT Length and NP

→ - Importance of Satisfying Hypothesis
  → - NP is an integer
    - Band-limited excitation

- Windowing

DFT Examples

Recall the theorem that provided for the relationship between the DFT terms and the Fourier Series Coefficients required

1.   The sampling window must be an integral number of periods

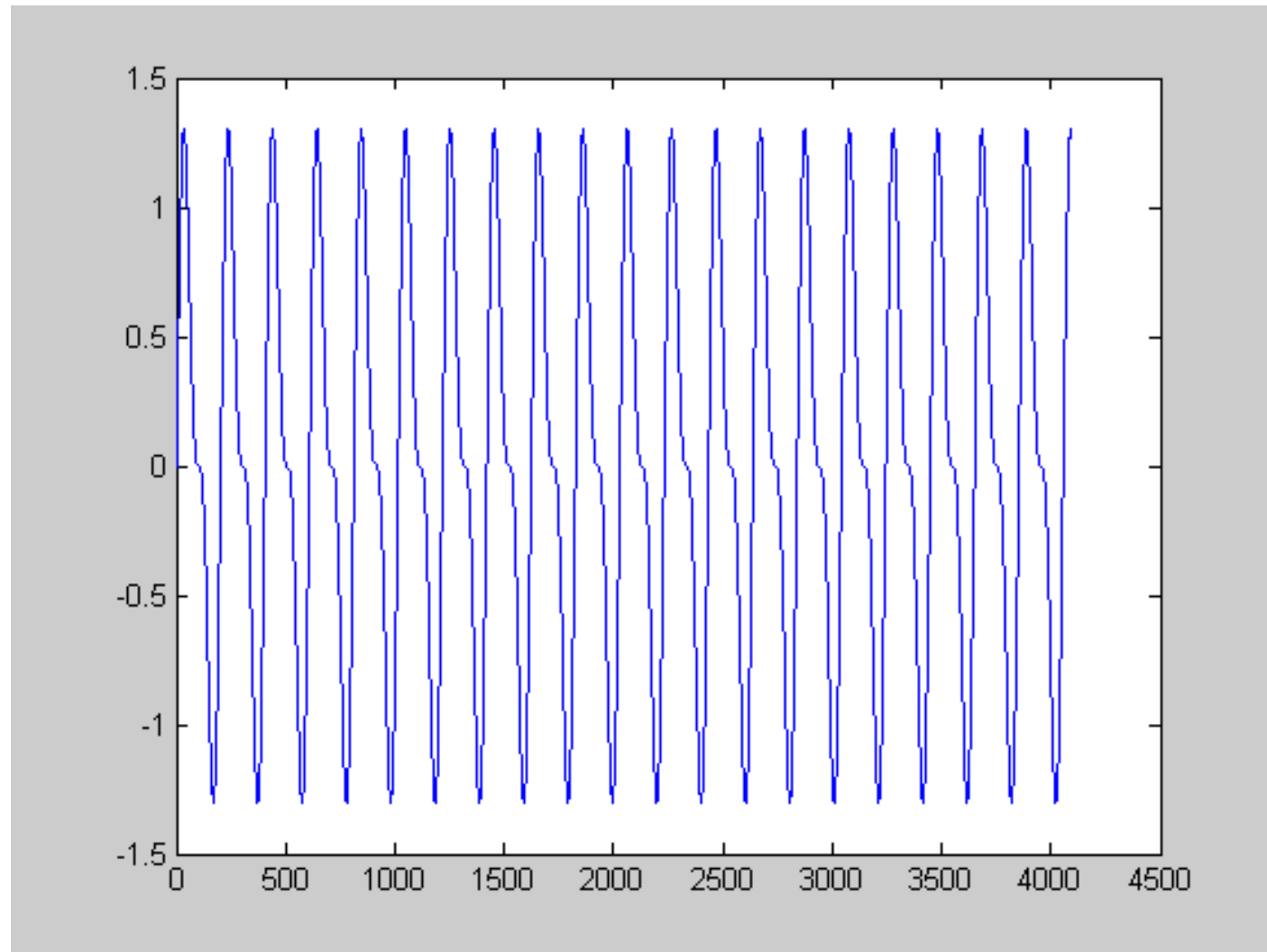2. $$N > \frac{2\,f_{max}}{f_{SIGNAL}} N_P$$

Example

WLOG assume f_{SIG}=50Hz

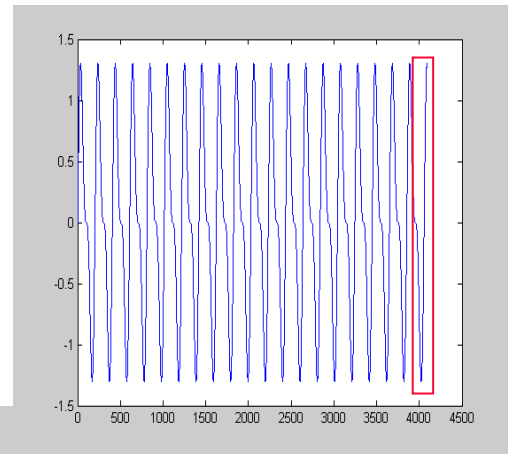$$V_{IN} = \sin(\omega t) + 0.5\sin(2\omega t)$$

$$\omega = 2\pi f_{SIG}$$

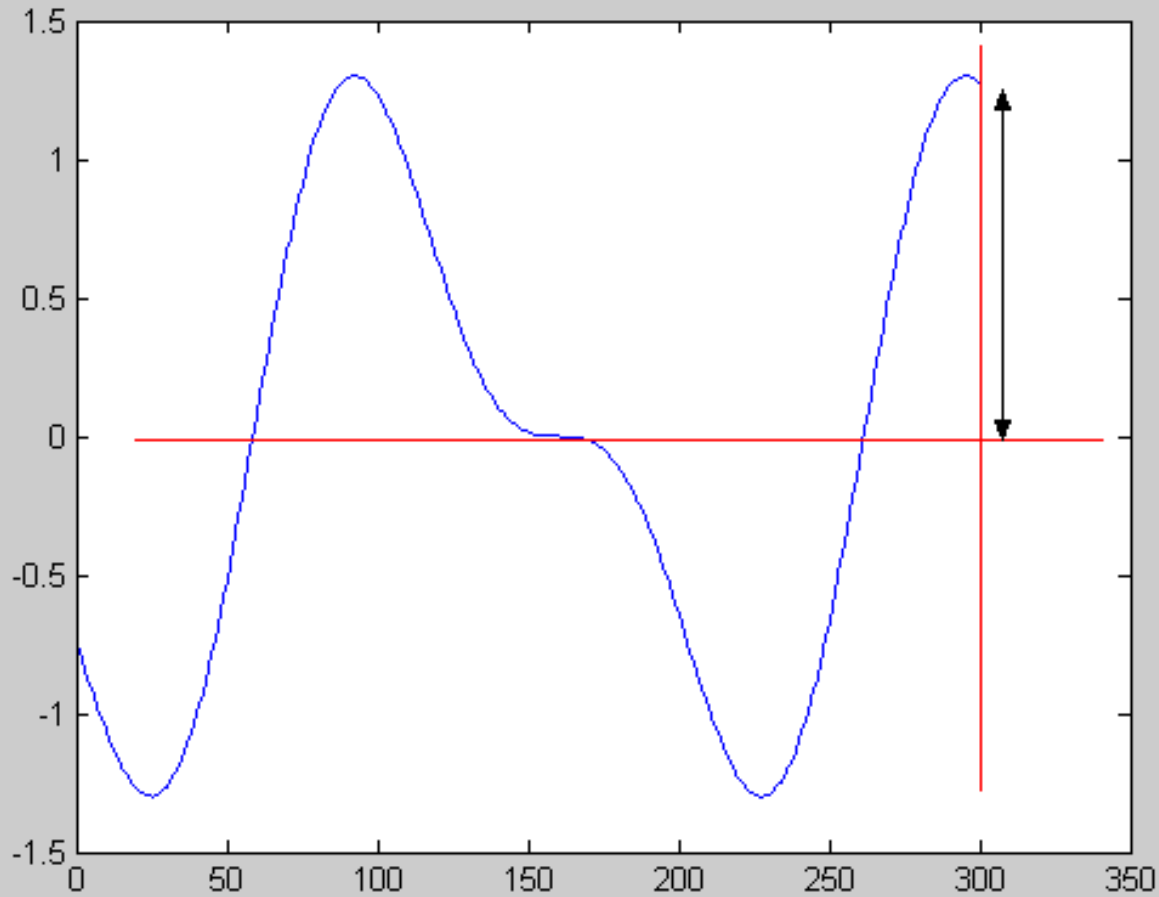Consider N_P=20.2 N=4096
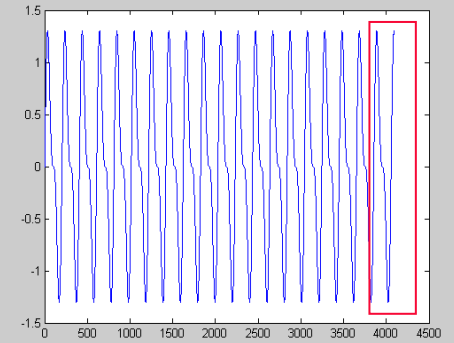
Recall $20\log_{10}(0.5) = -6.0205999$

Input Waveform
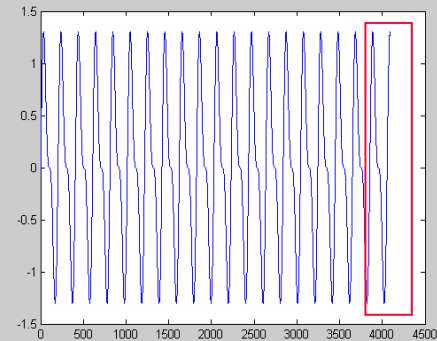
Input Waveform
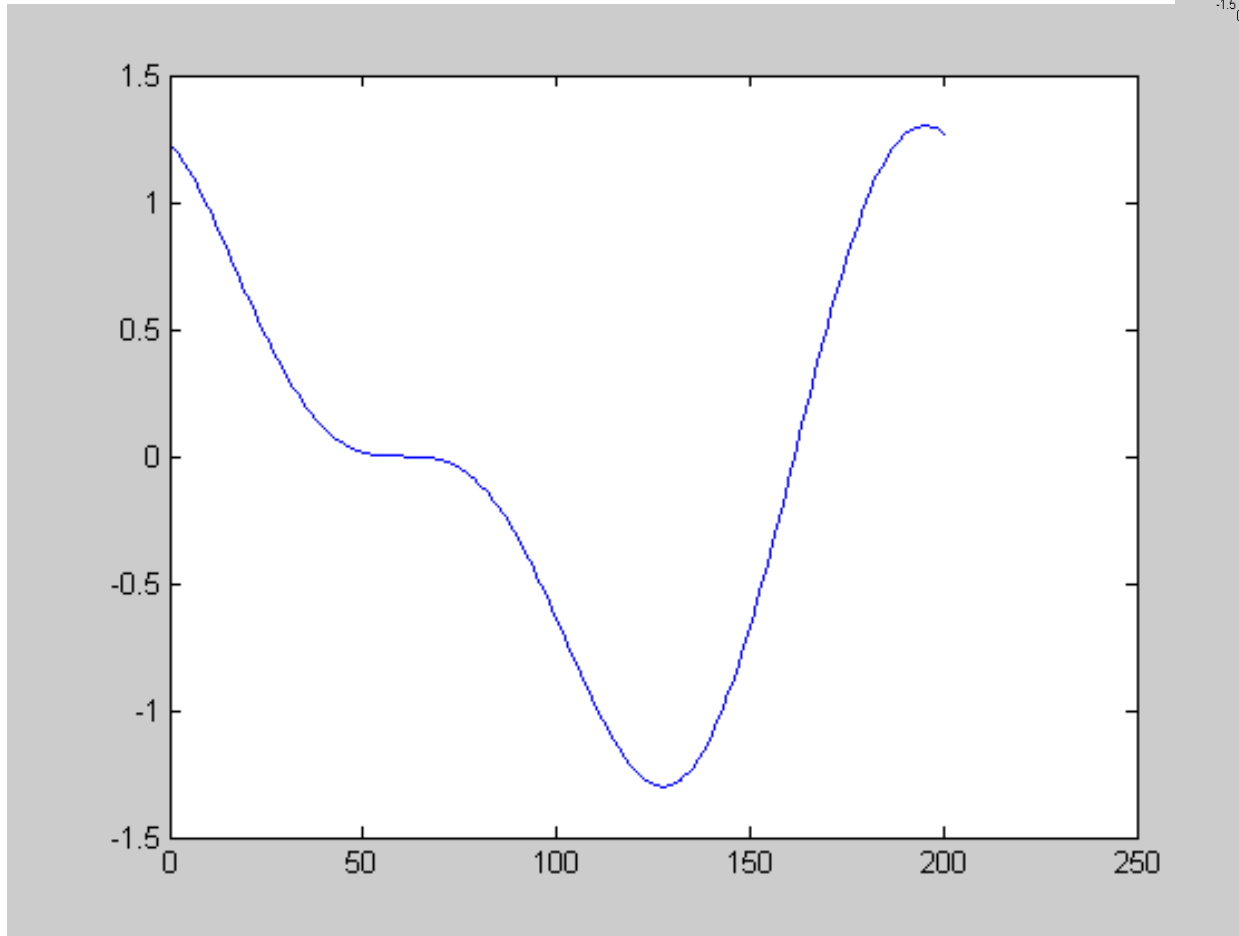
Input Waveform

Input Waveform

Spectral Response

Rect. Window   N=4096   Np =20.2
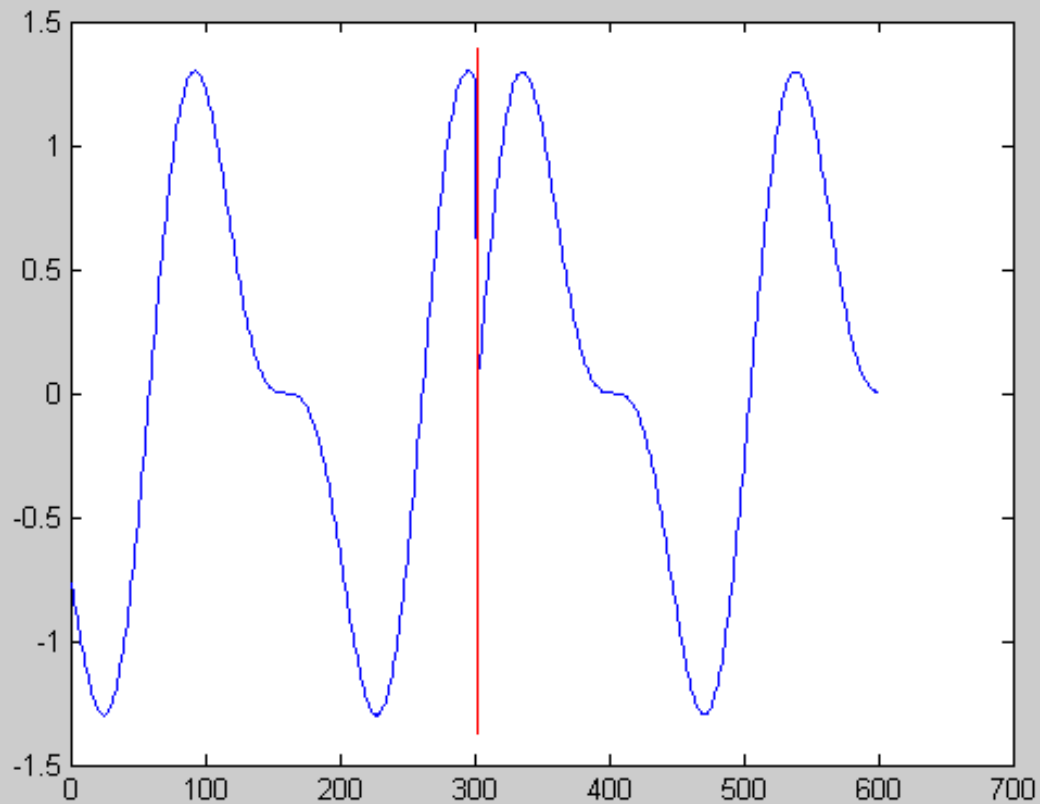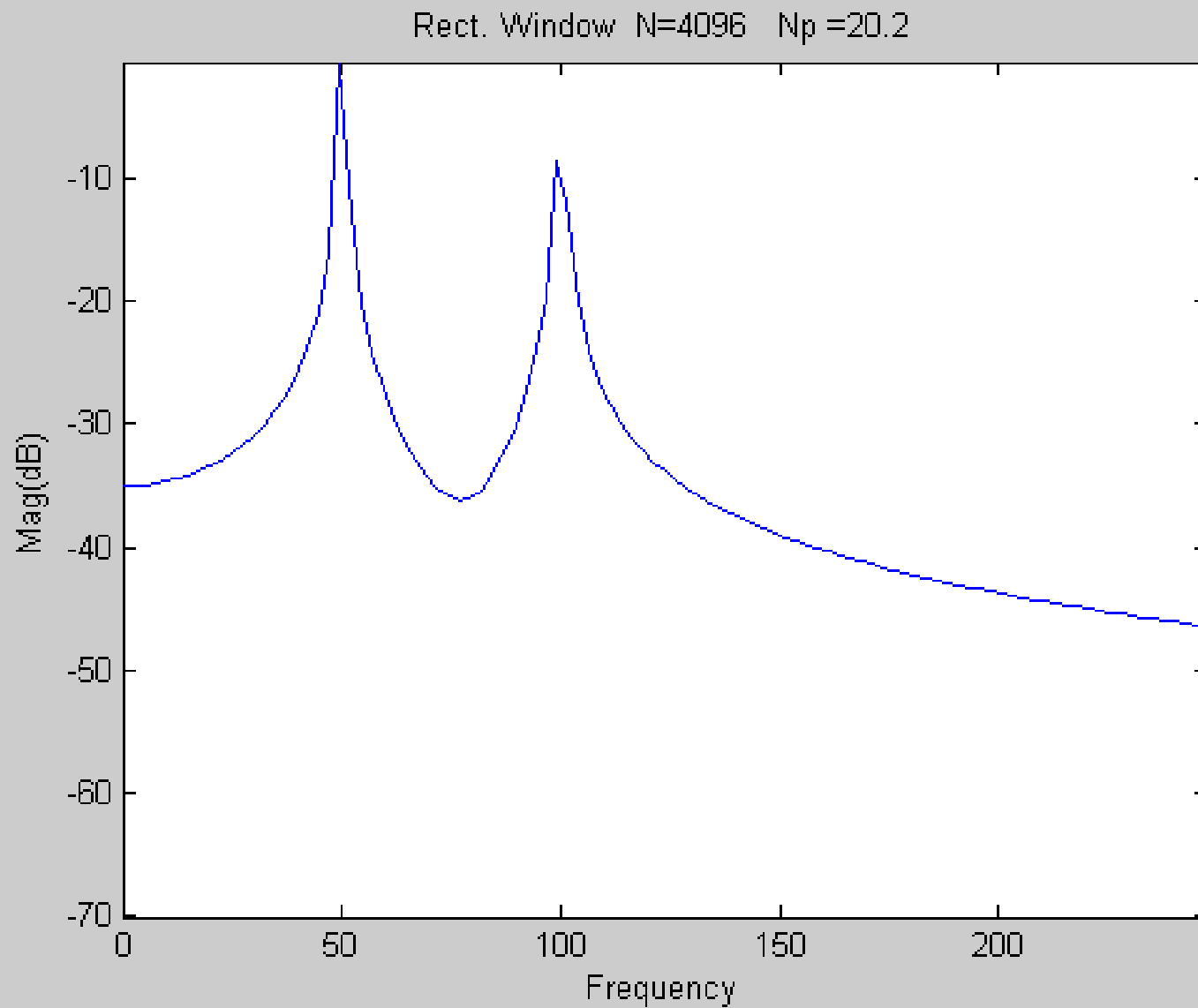
Fundamental will appear at position 1+Np = 21

Columns 1 through 7

-35.0366 -35.0125 -34.9400 -34.8182 -34.6458 -34.4208 -34.1403

Columns 8 through 14

-33.8005 -33.3963 -32.9206 -32.3642 -31.7144 -30.9535 -30.0563

Columns 15 through 21

-28.9855 -27.6830 -26.0523 -23.9155 -20.8888 -15.8561 **-0.5309**

Columns 22 through 28

-12.8167 -20.1124 -24.2085 -27.1229 -29.4104 -31.2957 -32.8782

Columns 29 through 35

-34.1902 -35.2163 -35.9043 -36.1838 -35.9965 -35.3255 -34.1946

Note there is a dramatic increase in the noise floor and a significant change in and spreading of the fundamental!!

kth harmonic will appear at position 1+k•Np

Columns 36 through 42

-32.6350 -30.6397 -28.1125 -24.7689 -19.7626 -8.5639 -11.7825

Columns 43 through 49

-20.0158 -23.9648 -26.5412 -28.4370 -29.9279 -31.1519 -32.1874

Columns 50 through 56

-33.0833 -33.8720 -34.5759 -35.2113 -35.7902 -36.3218 -36.8133

Columns 57 through 63

-37.2703 -37.6974 -38.0984 -38.4762 -38.8336 -39.1725 -39.4949

Columns 64 through 70

-39.8024 -40.0963 -40.3778 -40.6479 -40.9076 -41.1576 -41.3987

k[th] harmonic will appear at position 1+k•Np

Columns 36 through 42

-32.6350 -30.6397 -28.1125 -24.7689 -19.7626 ⟦-8.5639⟧ -11.7825

Columns 43 through 49

-20.0158 -23.9648 -26.5412 -28.4370 -29.9279 -31.1519 -32.1874

Columns 50 through 56

-33.0833 -33.8720 -34.5759 -35.2113 -35.7902 -36.3218 -36.8133

Columns 57 through 63

-37.2703 -37.6974 -38.0984 -38.4762 ⟦-38.8336⟧ -39.1725 -39.4949

Columns 64 through 70

-39.8024 -40.0963 -40.3778 -40.6479 -40.9076 -41.1576 -41.3987

Observations

- Modest change in sampling window of 0.2 out of 20 periods (1%) results in a big error in both fundamental and harmonic

- More importantly, dramatic raise in the noise floor !!! (from over -300dB to only -12dB)
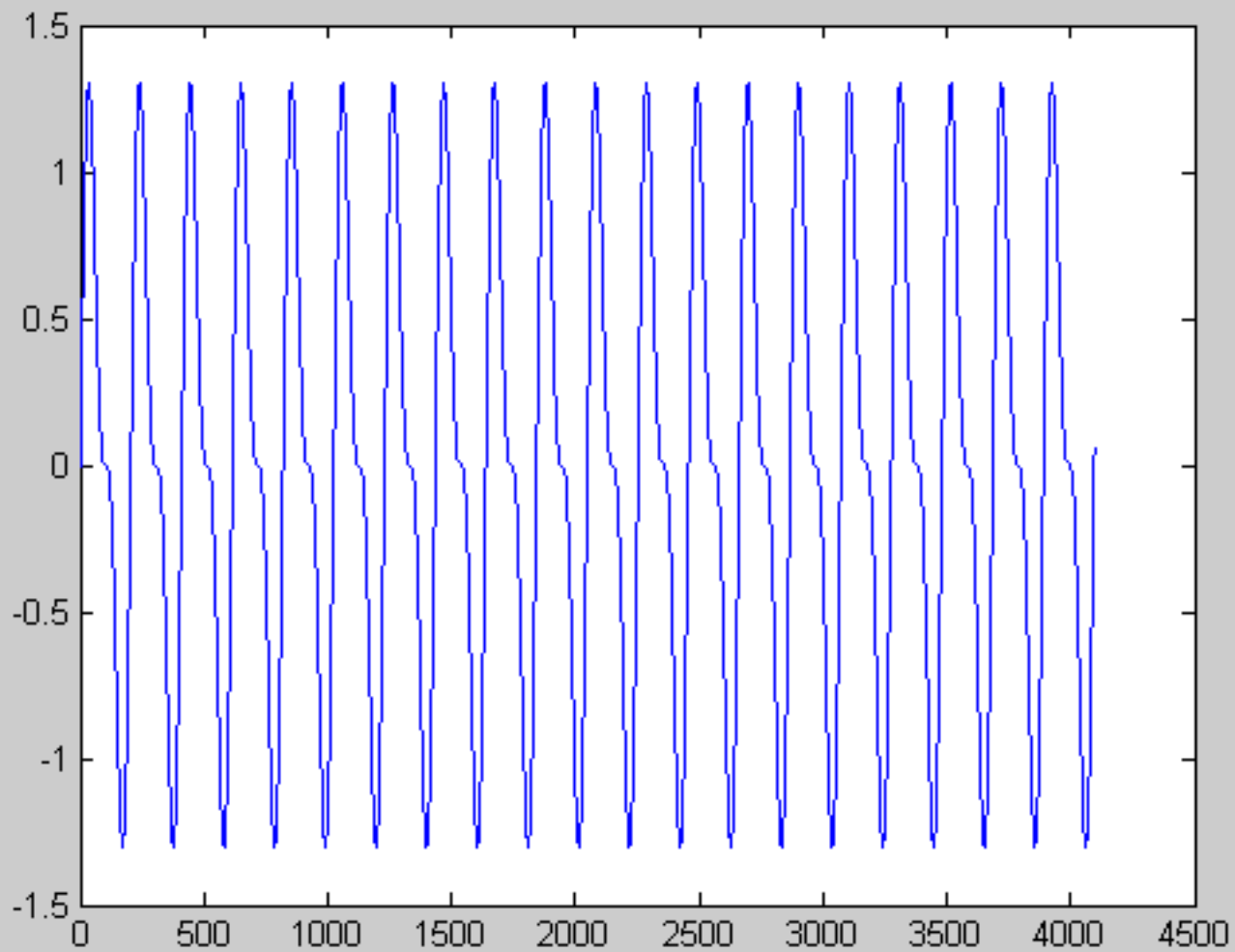
Example

WLOG assume f_{SIG}=50Hz

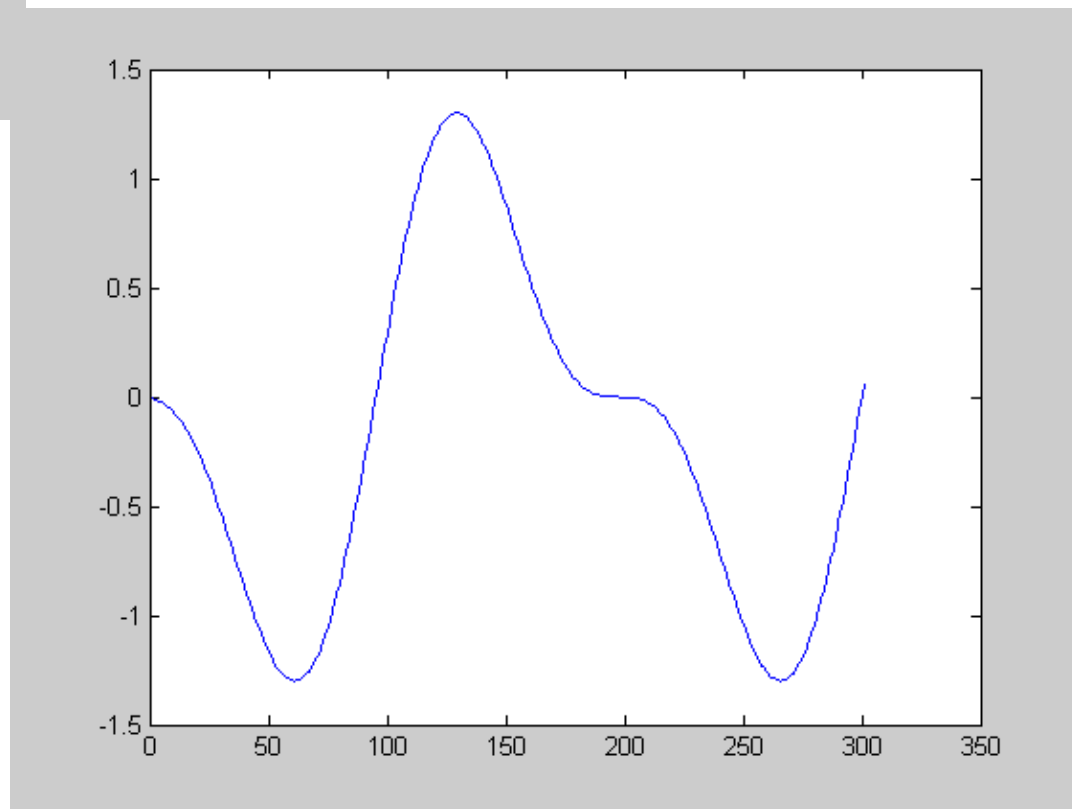$$V_{IN} = \sin(\omega t) + 0.5\sin(2\omega t)$$
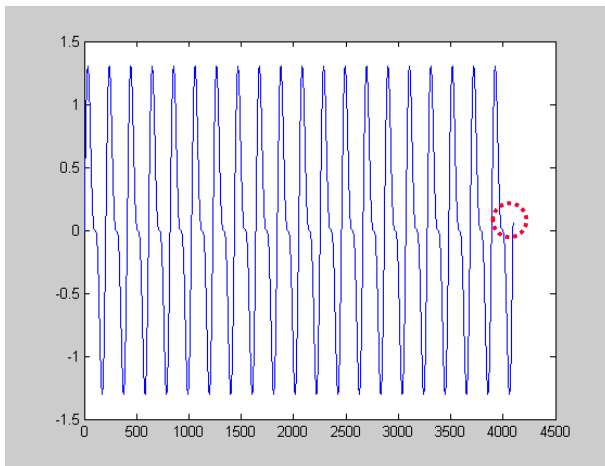
$$\omega = 2\pi f_{SIG}$$

Consider N_P=20.01 N=4096

Deviation from hypothesis is .05% of the sampling window

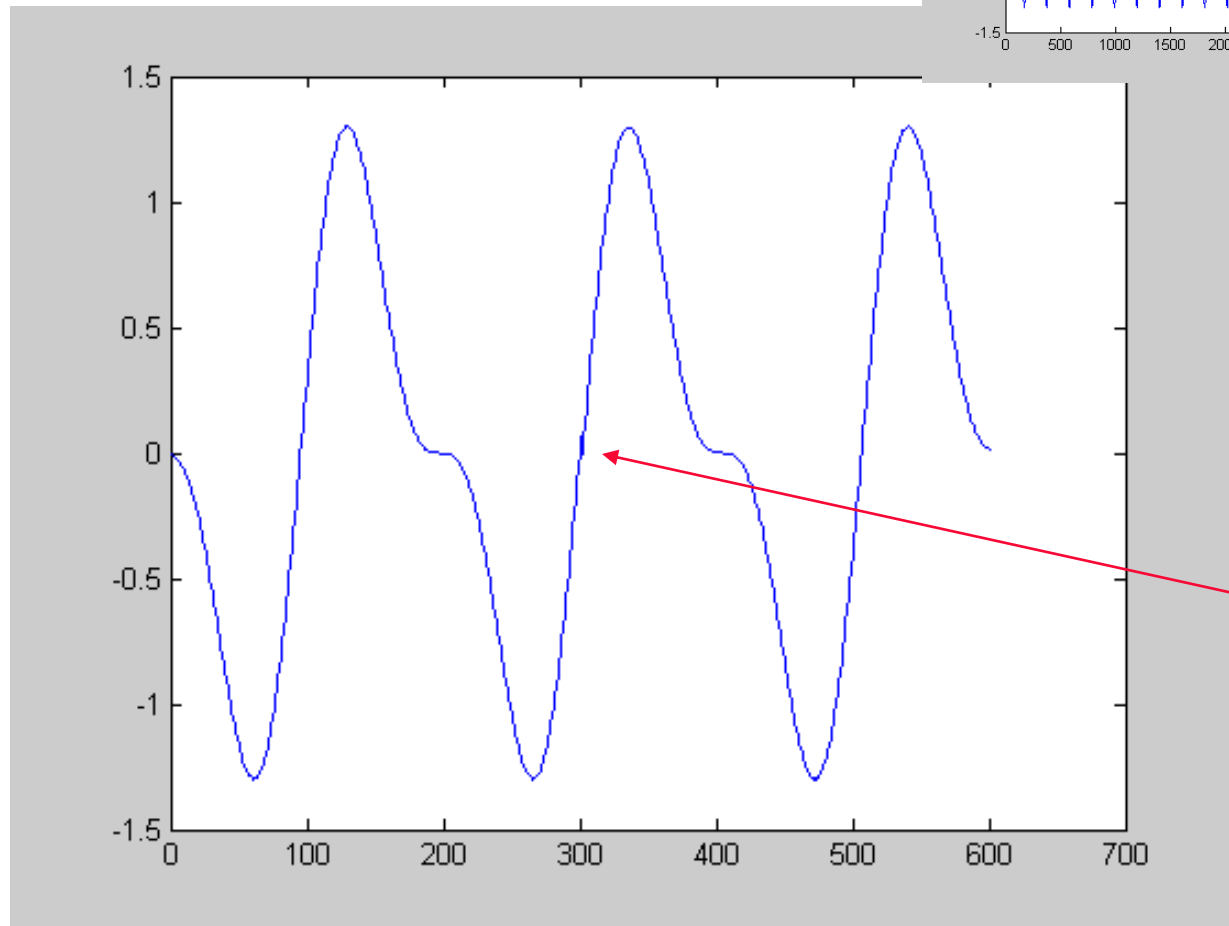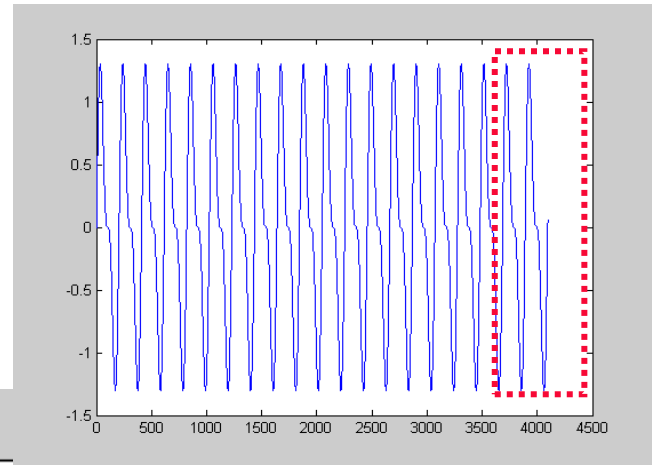# Input Waveform

Input Waveform

Input Waveform

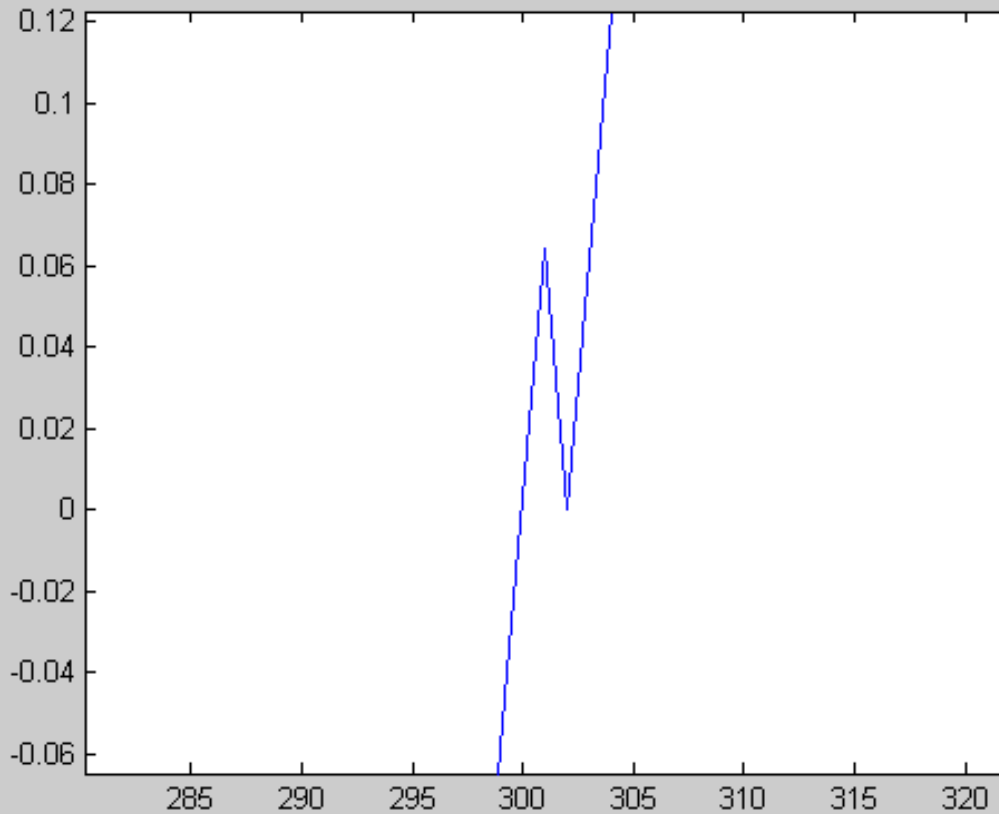# Input Waveform

Spectral Response with Non-Coherent Sampling

Rect. Window N=4096   Np =20.01

(zoomed in around fundamental)

Fundamental will appear at position 1+Np = 21

Columns 1 through 7

-89.8679 -83.0583 -77.7239 -74.2607 -71.6830 -69.5948 -67.8044

Columns 8 through 14

-66.2037 -64.7240 -63.3167 -61.9435 -60.5707 -59.1642 -57.6859

Columns 15 through 21

-56.0866 -54.2966 -52.2035 -49.6015 -46.0326 -40.0441 -0.0007

Columns 22 through 28

-40.0162 -46.2516 -50.0399 -52.8973 -55.3185 -57.5543 -59.7864

Columns 29 through 35

-62.2078 -65.1175 -69.1845 -76.9560 -81.1539 -69.6230 -64.0636

k^{th} harmonic will appear at position $1+k \cdot Np$

Columns 36 through 42

-59.9172 -56.1859 -52.3380 -47.7624 -40.9389 $\boxed{-6.0401}$ -39.2033

Observations

- Modest change in sampling window of 0.01 out of 20 periods (.05%) still results in a modest error in both fundamental and harmonic

- More importantly, substantial raise in the computational noise floor !!! (from over -300dB to only -40dB)

- Errors at about the 6-bit level !

Example

WLOG assume f_{SIG}=50Hz

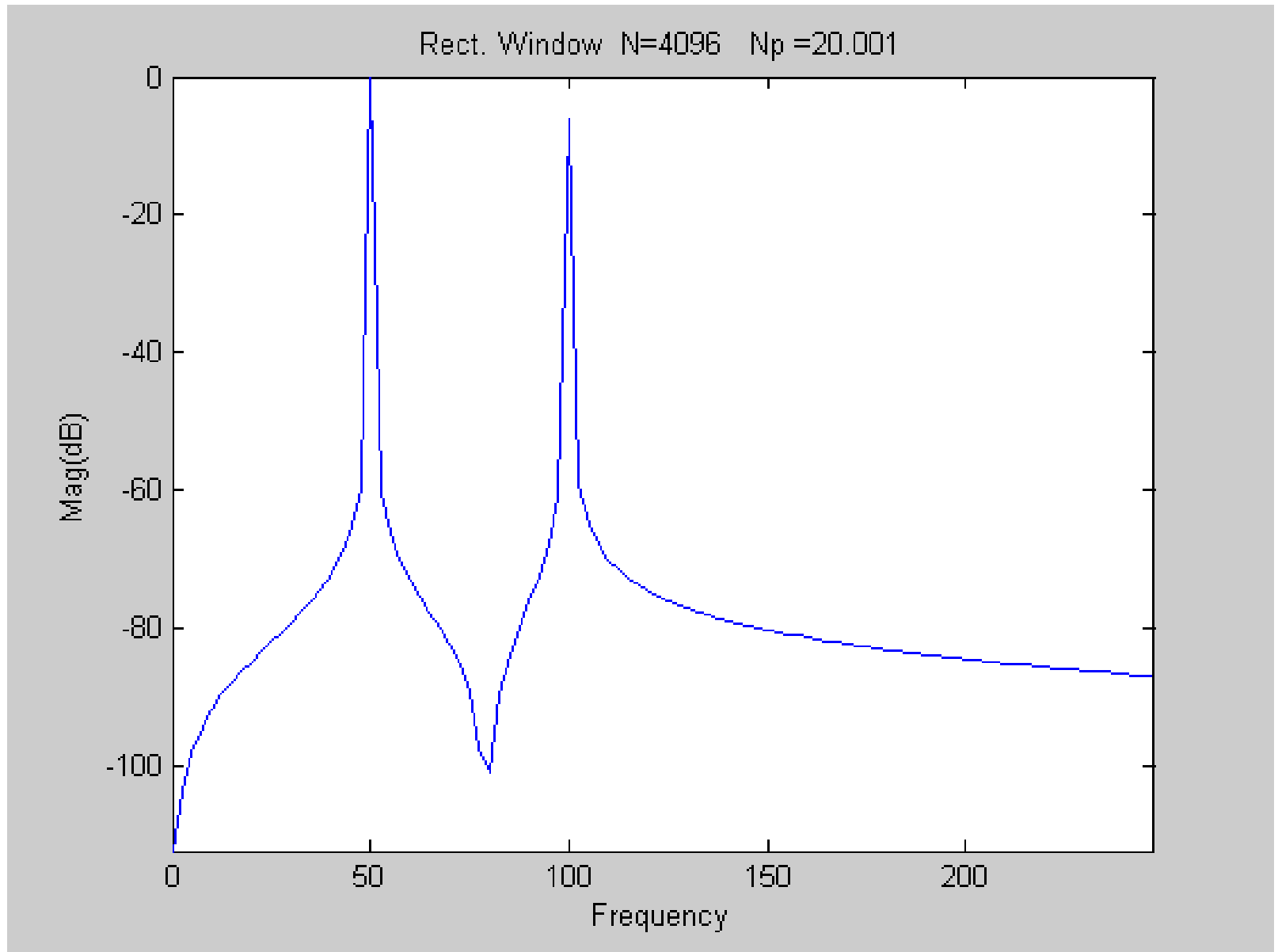$$V_{IN} = \sin(\omega t) + 0.5\sin(2\omega t)$$

$$\omega = 2\pi f_{SIG}$$

Consider N_P=20.001 N=4096

Deviation from hypothesis is .005% of the sampling window

Spectral Response with Non-coherent Sampling

(zoomed in around fundamental)

Fundamental will appear at position 1+Np = 21

Columns 1 through 7

-112.2531 -103.4507 -97.8283 -94.3021 -91.7015 -89.6024 -87.8059

Columns 8 through 14

-86.2014 -84.7190 -83.3097 -81.9349 -80.5605 -79.1526 -77.6726

Columns 15 through 21

-76.0714 -74.2787 -72.1818 -69.5735 -65.9919 -59.9650   0.0001

Columns 22 through 28

-60.0947 -66.2917 -70.0681 -72.9207 -75.3402 -77.5767 -79.8121

Columns 29 through 35

-82.2405 -85.1651 -89.2710 -97.2462 -101.0487 -89.5195 -83.9851

k[th] harmonic will appear at position 1+k•Np

Columns 36 through 42

-79.8472 -76.1160 -72.2601 -67.6621 -60.7642 -6.0220 -59.3448

Columns 43 through 49

-64.8177 -67.8520 -69.9156 -71.4625 -72.6918 -73.7078 -74.5718

Columns 50 through 56

-75.3225 -75.9857 -76.5796 -77.1173 -77.6087 -78.0613 -78.4809

Columns 57 through 63

-78.8721 -79.2387 -79.5837 -79.9096 -80.2186 -80.5125 -80.7927

Observations

- Modest change in sampling window of 0.01 out of 20 periods (.005%) results in a small error in both fundamental and harmonic

- More importantly, substantial raise in the computational noise floor !!! (from over -300dB to only -60dB)

- Errors at about the 10-bit level !

Spectral Response with Non-coherent sampling
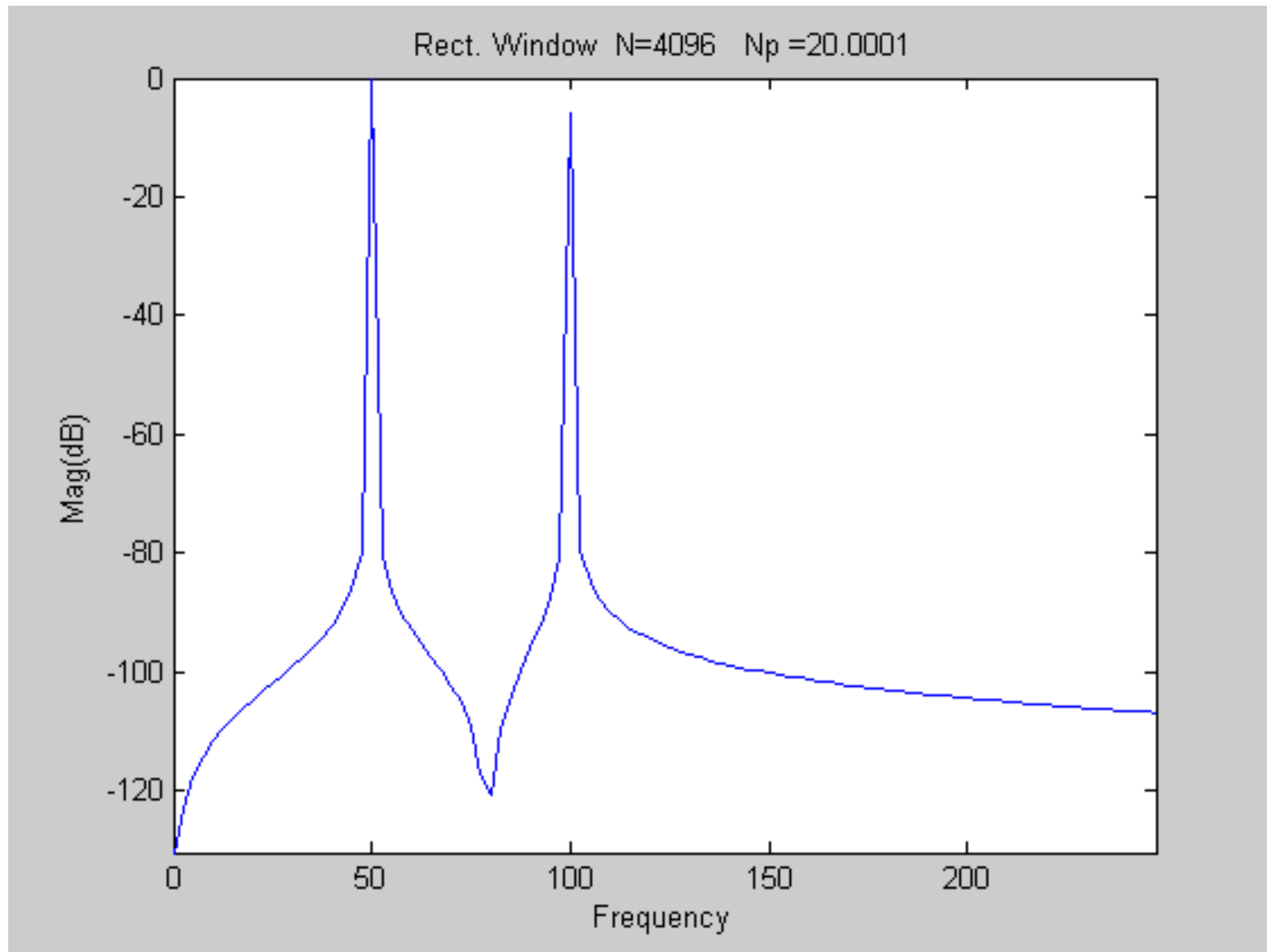
Rect. Window N=4096 Np =20.0001

(zoomed in around fundamental)

Fundamental will appear at position 1+Np = 21

Columns 1 through 7

-130.4427 -123.1634 -117.7467 -114.2649 -111.6804 -109.5888 -107.7965

 Columns 8 through 14

-106.1944 -104.7137 -103.3055 -101.9314 -100.5575 -99.1499 -97.6702

 Columns 15 through 21

-96.0691 -94.2764 -92.1793 -89.5706 -85.9878 -79.9571 | 0.0000 |

 Columns 22 through 28

-80.1027 -86.2959 -90.0712 -92.9232 -95.3425 -97.5788 -99.8141

 Columns 29 through 35

-102.2424 -105.1665 -109.2693 -117.2013 -120.8396 -109.4934 -103.9724

k^{th} harmonic will appear at position $1+k \cdot Np$

Columns 36 through 42

-99.8382 -96.1082 -92.2521 -87.6522 -80.7470 | -6.0207 | -79.3595

Columns 43 through 49

-84.8247 -87.8566 -89.9190 -91.4652 -92.6940 -93.7098 -94.5736

Columns 50 through 56

-95.3241 -95.9872 -96.5810 -97.1187 -97.6100 -98.0625 -98.4821

Columns 57 through 63

-98.8732 -99.2398 -99.5847 -99.9107 | -100.2197 | -100.5135 -100.7937

Columns 64 through 70

Observations

- Modest change in sampling window of 0.001 out of 20 periods (.0005%) results in a small error in both fundamental and harmonic

- More importantly, substantial raise in the computational noise floor !!! (from over -300dB to only -80dB)

- Errors at about the 13-bit level !

Stay Safe and Stay Healthy !

End of Lecture 28